Similar and Congruent Figures

1. For each pair of triangles, state whether they are **congruent**, **similar but not congruent**, or **neither**. If congruent, name the condition (SSS, SAS, ASA, AAS, or HS).

   a)

   $\triangle ABC$ has sides $AB = 5, BC = 7, CA = 9.$
   $\triangle DEF$ has sides $DE = 10, EF = 14, FD = 18.$

   $$\frac{AB}{DE} = \frac{5}{10} = \frac{1}{2}, \quad \frac{BC}{EF} = \frac{7}{14} = \frac{1}{2}, \quad \frac{CA}{FD} = \frac{9}{18} = \frac{1}{2}$$

   All three ratios are equal, so the corresponding sides are proportional. The triangles are **similar** but **not congruent** (the sides are not equal, only proportional).

   b)

   $\triangle PQR$ has $PQ = 6, \angle P = 42°, PR = 8.$
   $\triangle XYZ$ has $XY = 6, \angle X = 42°, XZ = 8.$

   $\triangle PQR$ has $PQ = 6, \angle P = 42°, PR = 8.$ $\triangle XYZ$ has $XY = 6, \angle X = 42°, XZ = 8.$

   Both triangles have two sides of equal length (6 and 8) with the equal angle ($42°$) **between** those sides. This matches the **SAS** condition.

   $$\therefore \triangle PQR \cong \triangle XYZ \text{ (SAS)}$$

   c)

   $\triangle GHI$ has $\angle G = 55°, \angle H = 75°, GH = 12.$
   $\triangle JKL$ has $\angle J = 55°, \angle K = 75°, JK = 12.$

   $\triangle GHI$ has $\angle G = 55°, \angle H = 75°, GH = 12.$ $\triangle JKL$ has $\angle J = 55°, \angle K = 75°, JK = 12.$

   Both triangles have two equal angles ($55°$ and $75°$) with the equal side (12) **between** those angles. This matches the **ASA** condition.

   $$\therefore \triangle GHI \cong \triangle JKL \text{ (ASA)}$$

2. The two triangles below are **similar**. Find the value of x.

   $\triangle ABC$ has sides $AB = 4$, $BC = x$, $CA = 6$.

   $\triangle DEF$ has sides $DE = 10$, $EF = 15$, $FD = 15$.

   HINT: Drawing a rough diagram may help you build the ratio of similarity.

$$\frac{AB}{DE} = \frac{4}{10} = \frac{2}{5}$$

Verify with the other known pair:

$$\frac{CA}{FD} = \frac{6}{15} = \frac{2}{5}\checkmark$$

Now use the ratio to find x:

$$\frac{BC}{EF} = \frac{2}{5}$$

$$\frac{x}{15} = \frac{2}{5}$$

$$x = 15 \times \frac{2}{5} = 6$$

$$\therefore x = 6$$

3. A student wants to estimate the height of a flagpole.

   She holds a metre stick vertically, and it casts a shadow 0.8 m long at the same time the flagpole casts a shadow 6.4 m long.

   The two triangles formed (metre stick + its shadow, flagpole + its shadow) are **similar**.

   Find the height of the flagpole.

$$\frac{\text{height of metre stick}}{\text{height of flagpole}} = \frac{\text{shadow of metre stick}}{\text{shadow of flagpole}}$$

   Let h be the height of the flagpole in metres.

   The metre stick (height 1 m) casts a shadow of 0.8 m; the flagpole casts a shadow of 6.4 m.

   Using the given proportion:

$$\frac{\text{height of metre stick}}{\text{height of flagpole}} = \frac{\text{shadow of metre stick}}{\text{shadow of flagpole}}$$

$$\frac{1}{h} = \frac{0.8}{6.4}$$

   Cross-multiply:

$$0.8h = 1 \times 6.4$$

$$0.8h = 6.4$$

$$h = \frac{6.4}{0.8} = 8$$

   $\therefore$ The flagpole is 8 m tall.