Triangle Centres

Circumcentre

The *circumcentre* is the point where the three perpendicular bisectors of a triangle meet.

To see this:

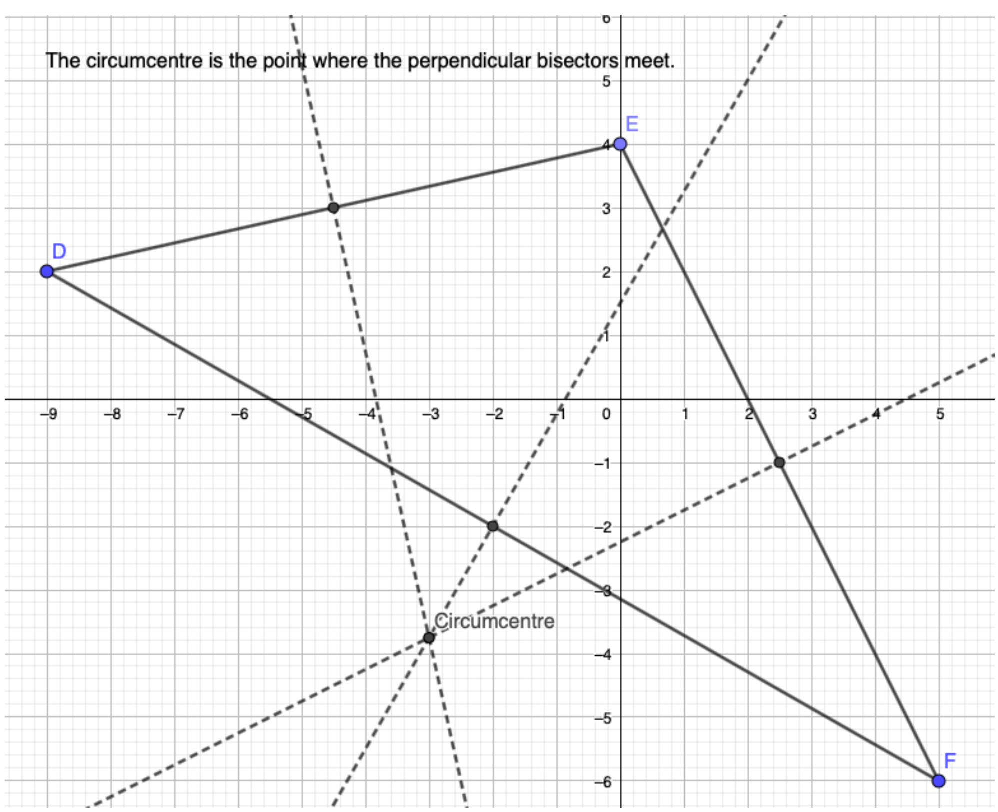
a. Use Geogebra Classic to plot the triangle defined by the points $D(-9,\ 2)$, $E(0,\ 4)$, and $F(5,\ -6)$.
b. Construct the midpoints of each side of the triangle.
c. Construct the three perpendicular bisectors. Change the line style so they are dashed.
d. Construct the circumcentre. Select the point, then type *Circumcentre*.
e. Now pick one of the perpendicular bisectors. In point form, describe: using algebra, how could you find the equation that defines this line?

· get slope of DE; slope of perpendicular bisector is negative reciprocal
· use midpoint and that slope to get vertical intercept of perpendicular bisector
· now we have it's equation

f. If you had the equation for two perpendicular bisectors, how could you use algebra to determine where they meet?

· use linear systems (elimination or substitution) to obtain circumcentre (their point of intersection)

g. Reproduce your Geogebra construction in the space below. Use a ruler.



Orthocentre

The *orthocentre* is the point where the three altitudes of a triangle meet. To see this:

a. Use Geogebra Classic to plot the triangle defined by the points $A(8,\ 2)$, $B(-3,\ 5)$, and $C(5,\ 11)$.

b. Construct all three altitudes. Change the line style so they are dashed.

c. Construct the orthocentre. Select the point, then type *Orthocentre*.

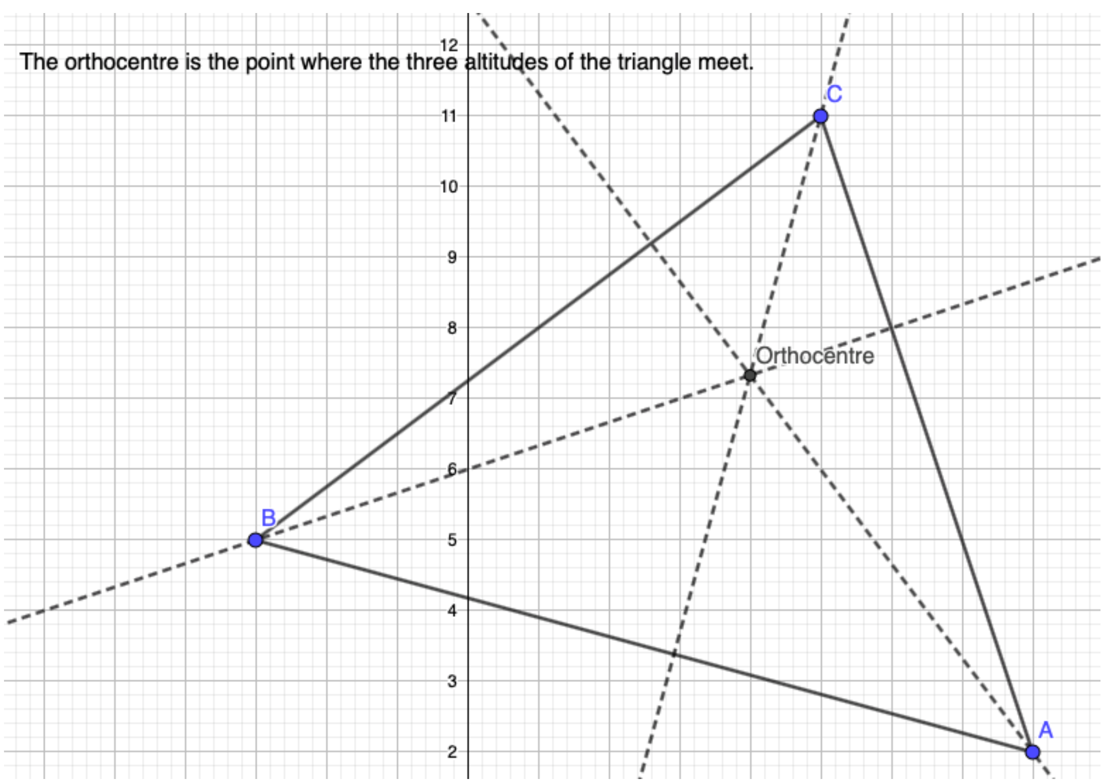
d. Now pick one of the altitudes. In point form, describe: using algebra, how could you find the equation that defines this line?

- get slope of AB; slope of altitude, since it is perpendicular, is the negative reciprocal
- use that slope and point C to get vertical intercept
- now you have equation for altitude

e. If you had the equation for two altitudes, how could you use algebra to determine where they meet?

- use linear systems (elimination or substitution) to obtain orthocentre (their point of intersection)

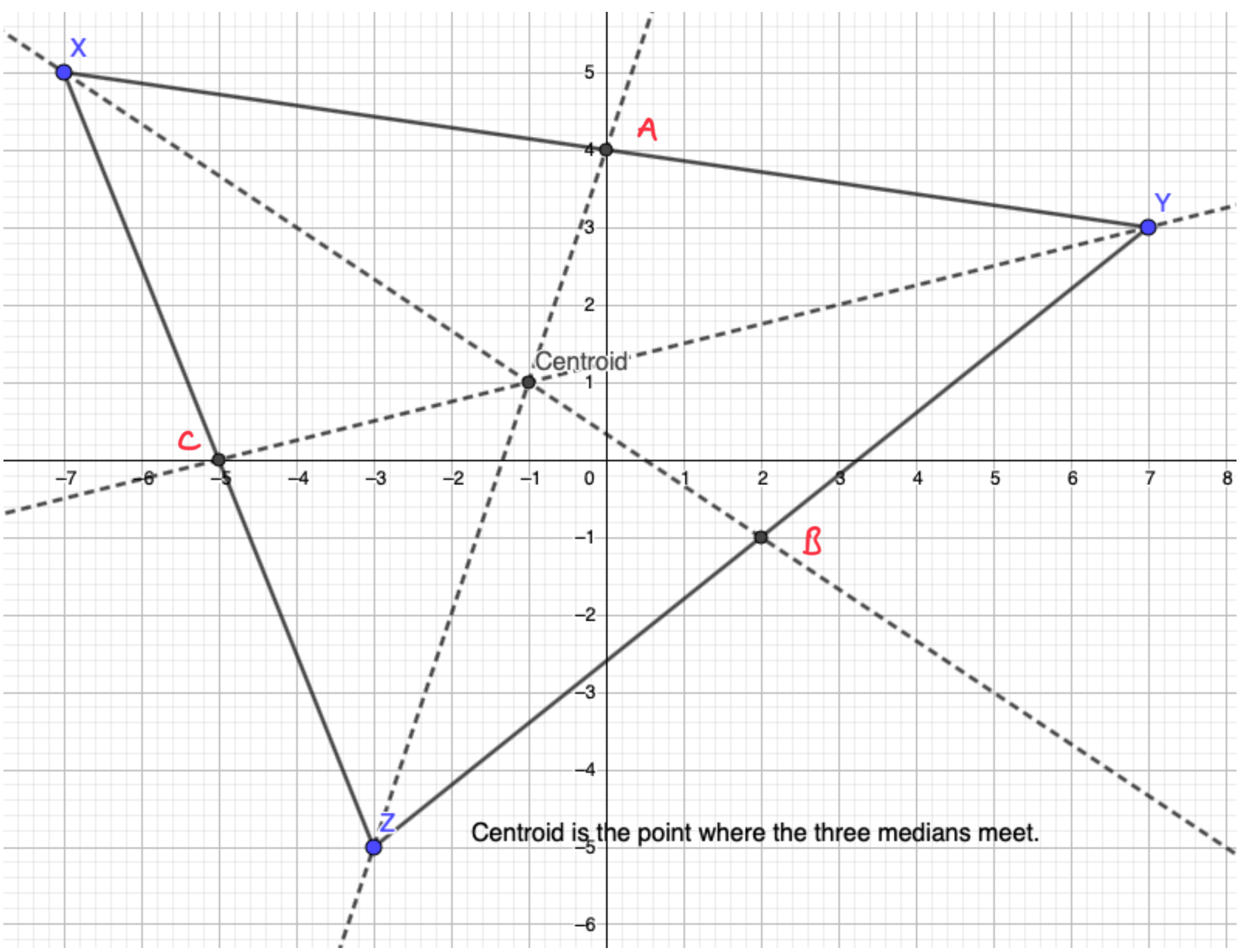
f. Reproduce your Geogebra construction in the space below. Use a ruler.



Centroid

The *centroid* is the point where the three medians of a triangle meet. To see this:

a. Use Geogebra Classic to plot the triangle defined by the points $X(-7,\ 5)$, $Y(7,\ 3)$, and $Z(-3,\ -5)$.
b. Construct the midpoints of each side of the triangle.
c. Construct the three medians. Change the line style so they are dashed.
d. Construct the centroid. Select the point, then type *Centroid*.
e. Now pick one of the medians. In point form, describe: how could you find the equation that defines this line?
   - get midpoint, A, of line segment XY
   - use slope formula with points A and Z to get slope
   - use newfound slope and point A to get vertical intercept
   - now you have equation for a median
f. If you had the equation for two medians, how could you use algebra to determine where they meet?
   use linear systems (elimination or substitution) to obtain centroid (their point of intersection)
g. Reproduce your Geogebra construction in the space below. Use a ruler.



Practice

Go back to one of the three triangle centres noted above (your choice).

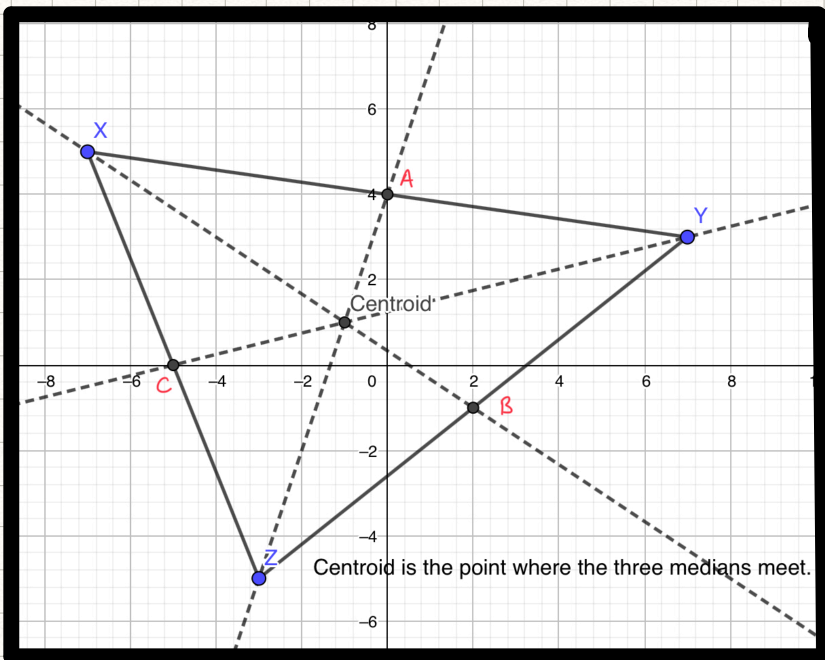
Find the exact co-ordinates of the triangle centre noted.

Check your work by comparing to your Geogebra sketch.

(see next page for example)

Centroid

$X(-7, 5) \quad Y(7, 3) \quad Z(-3, -5)$



get midpoints

$$M_{XY} = \left(\frac{-7+7}{2}, \frac{5+3}{2}\right) \qquad M_{YZ} = \left(\frac{7+(-3)}{2}, \frac{3+(-5)}{2}\right) \qquad M_{XZ} = \left(\frac{-7+(-3)}{2}, \frac{5+(-5)}{2}\right)$$

$$= \left(\frac{0}{2}, \frac{8}{2}\right) \qquad = \left(\frac{4}{2}, \frac{-2}{2}\right) \qquad = \left(\frac{-10}{2}, \frac{0}{2}\right)$$

$$A = (0, 4) \qquad B = (2, -1) \qquad C = (-5, 0)$$

get slopes of two median lines

$$m_{AZ} = \frac{4-(-5)}{0-(-3)} = \frac{9}{3} = 3 \qquad m_{XB} = \frac{-1-5}{2-(-7)} = \frac{-6}{9} = -\frac{2}{3}$$

get vertical intercepts of two median lines

For AZ: $4 = (3)(0) + b$

$4 = b$

For XB: $-1 = \left(-\frac{2}{3}\right)(2) + b$

$-\frac{3}{3} = -\frac{4}{3} + b$

$+\frac{4}{3} \qquad +\frac{4}{3}$

$\frac{1}{3} = b$

state equation of each median line

AZ equation is: $y = 3x + 4$

XB equation is: $y = -\frac{2}{3}x + \frac{1}{3}$

find intersection point (centroid) using substitution

$$y = 3x + 4$$
$$y = -\frac{2}{3}x + \frac{1}{3}$$

Sub one equation's y-value into another to obtain x-value:

$$3x + 4 = -\frac{2}{3}x + \frac{1}{3}$$
$$+\frac{2}{3}x \quad -4 \quad +\frac{2}{3}x \quad -4$$
$$\frac{9}{3}x + \frac{2}{3}x = \frac{1}{3} - \frac{12}{3}$$
$$\frac{11}{3}x = -\frac{11}{3}$$
$$\frac{\frac{11}{3}x}{\frac{11}{3}} = \frac{-\frac{11}{3}}{\frac{11}{3}}$$
$$x = -1$$

Sub x into an equation to get y-value:

$$y = 3x + 4$$
$$y = 3(-1) + 4$$
$$y = -3 + 4$$
$$y = 1$$

$\therefore$ the point of intersection is $(-1, 1)$. This represents the centroid. I have verified that my algebraic work is correct by consulting my Geogebra sketch.