MPM2D Class #: _____ Date: ______________

Transformations of the Quadratic Relation

Recall The "basic" quadratic relation looks like this:

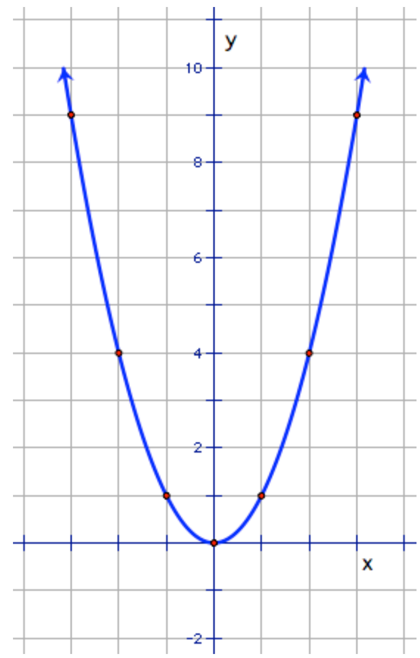
<u>In symbolic form</u> <u>In a table</u> <u>In graphical form</u>

$y = x^2$

$y = 1(x-0)^2 + 0$

a h k

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



Conclusions from Modeling

Summarize your results from earlier in this unit.

When a value is added to the basic quadratic relation, for example with:

$y = x^2 + 2$ … then the graph is translated up 2 units

When a value is subtracted from the basic quadratic relation, for example, with:

$y = x^2 - 5$ … then the graph is translated down 5 units.

By convention we call this value k. So with the quadratic relation in symbolic form:

$y = x^2 + k$

… when k is positive, the graphical form will be translated up

… when k is negative, the graphical form will be translated down

A quadratic relation can also be in the form:

NOTE: If $h = -2$ then…

$$y = (x-h)^2$$

$y = (x-h)^2$
$y = (x-(-2))^2$
$y = (x+2)^2$

… where an h value is subtracted inside the brackets.

So the h value in an equation is "tricky".

For example, if the relation is:

$$y = (x-3)^2$$ … then the graph is translated right by 3 units.

$h=3$

Actual sign of h value is opposite of what it appears to be.

If the relation is:

$$y = (x+2)^2$$ … then the graph is translated left by 2 units.

$h=-2$

So with the quadratic relation in symbolic form:

$$y = (x-h)^2$$

… when h is positive, the graphical form will be translated right

… when h is negative, the graphical form will be translated left

Example 1

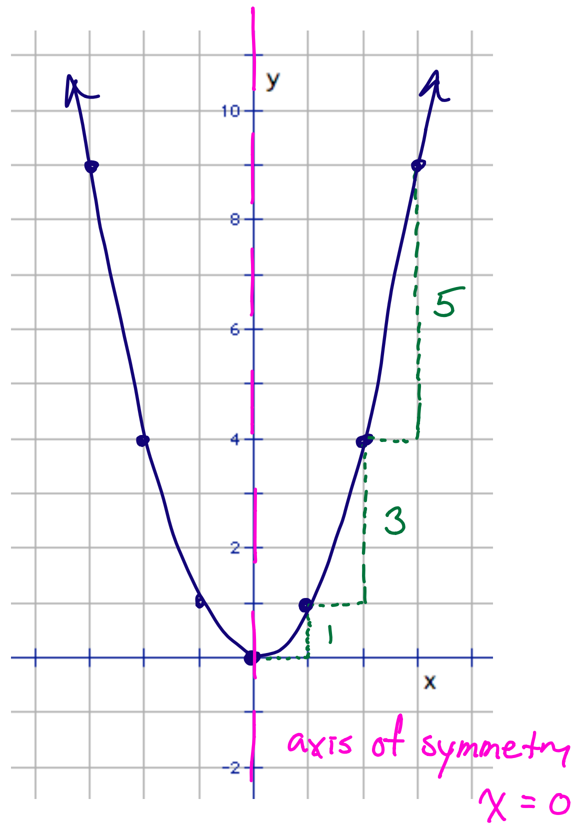
Graph the basic quadratic relation $y = x^2$.

$$y = \underset{a}{1}(x - \underset{h}{0})^2 + \underset{k}{0}$$

Note that the table of values is:

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

first differences
1
3
5



Start by plotting the vertex.

Plot the points on the right side using the step pattern.

Then reflect the points in the y-axis to get the "other half". (We can do this because parabolas are symmetrical).

a) What is the direction of opening?

up

b) What are the co-ordinates of the vertex?

(0, 0)

c) What is the equation of the axis of symmetry?

$x = 0$

d) What is the minimum value?

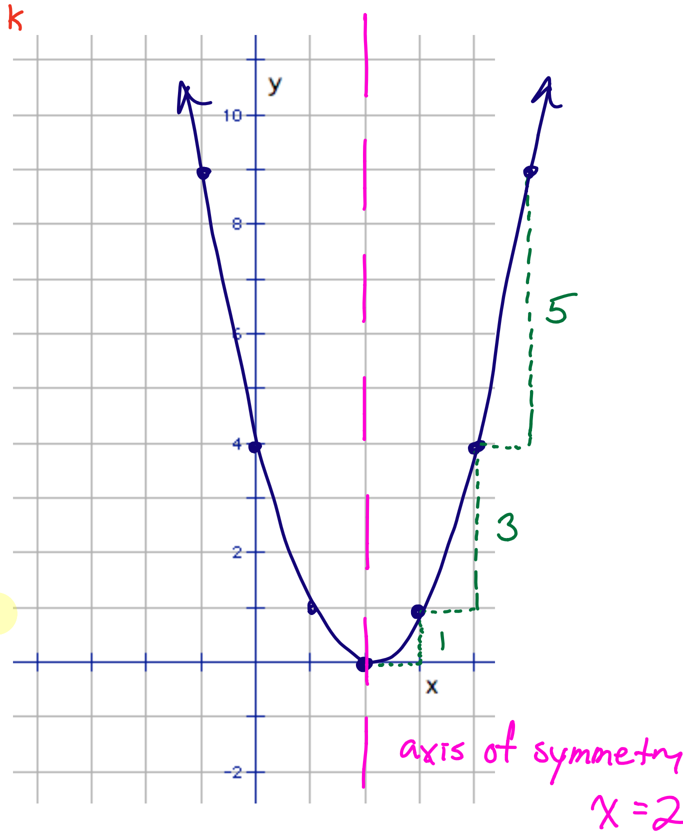
minimum value is 0.

Example 2

$y = 1(x-2)^2 + 0$

a h k

Graph the quadratic relation $y = (x-2)^2$.



We still start by graphing the vertex.

Where is the vertex? (2 , 0) .

h k

Next use the same step-pattern (1, 3, 5, …) to plot the points on the right side.

Then reflect the points in the y-axis to get the "other half".

Note that our plotted points, using the "step pattern process" are correct for the quadratic relation $y = (x-2)^2$ when we look at it in tabular form:

x	y
-1	9
0	4
1	1
2	0
3	1
4	4

a) What is the direction of opening?

up

b) What are the co-ordinates of the vertex?

(2, 0)

c) What is the equation of the axis of symmetry?

$x = 2$

d) What is the minimum value?

minimum value is 0

Example 3

$y=1(x-(-1))^2-2$

a h k

Graph the quadratic relation $y=(x+1)^2-2$.

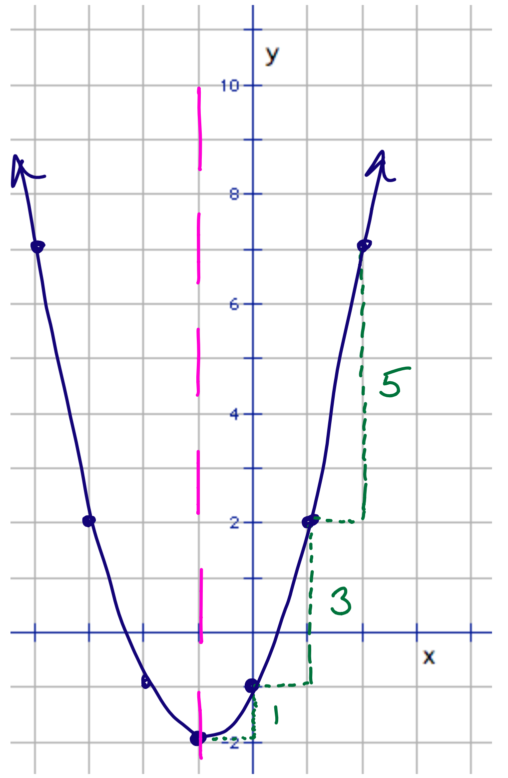
We still start by graphing the vertex.

Where is the vertex? $(-1,-2)$.

h k

Use the same step pattern (1, 3, 5…), then reflect the points in the y-axis to get the "other half".

Again, note that our plotted points are still correct, when compared to the table of values:



x	y
-4	7
-3	2
-2	-1
-1	-2
0	-1
1	2
2	7

… except that plotting with the "step pattern" is much faster than directly computing the table of values.

axis of symmetry

$x=-1$

a) What is the direction of opening?

up

b) What are the co-ordinates of the vertex?

$(-1,-2)$

c) What is the equation of the axis of symmetry?

$x=-1$

d) What is the minimum value?

minimum value is -2

Opportunity to Learn

All visible questions on the next page. We will take these up in our next class. You may use Desmos to check your work.

2. Sketch graphs of these three quadratic relations on the same set of axes.

a) $y = (x - 9)^2$

b) $y = (x + 2)^2$

c) $y = (x - 5)^2$

3. Sketch graphs of these three quadratic relations on the same set of axes.

a) $y = x^2 + 8$

b) $y = x^2 - 5$

c) $y = x^2 - 10$

4. Sketch the graph of each parabola. Label at least three points on the parabola. Describe the transformation from the graph of $y = x^2$.

c) $y = x^2 - 5$

d) $y = (x - 8)^2$

e) $y = -\frac{1}{2}x^2$

g) $y = x^2 + 0.5$

Connect and Apply

6. Write an equation for the quadratic relation that results from each transformation.

a) The graph of $y = x^2$ is translated 6 units upward.

b) The graph of $y = x^2$ is translated 4 units downward.

7. Write an equation for the quadratic relation that results from each transformation.

a) The graph of $y = x^2$ is translated 7 units to the left.

b) The graph of $y = x^2$ is translated 5 units to the right.

c) The graph of $y = x^2$ is translated 8 uni to the left.

d) The graph of $y = x^2$ is translated 3 units to the right.

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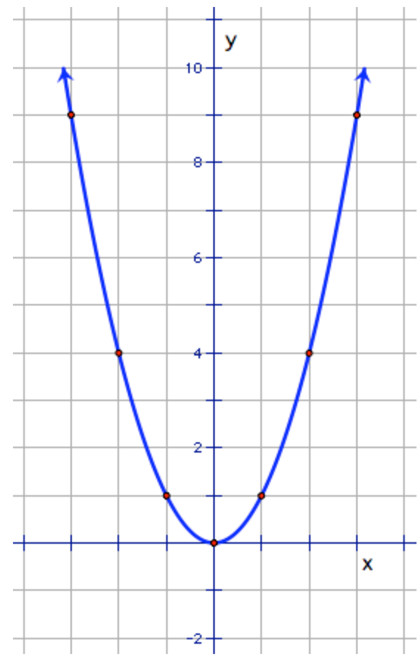
More Transformations of the Quadratic Function

Recall The "basic" quadratic function looks like this:

<u>In symbolic form</u> <u>In a table</u> <u>In graphical form</u>

$y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



Conclusions from Investigations

Summarize your results from earlier in this unit.

When the basic quadratic relation is multiplied by a value greater than one, for example with:

$y = 2x^2$ … then the graph is stretched* .

*formally: "scale factor of 2 creating a stretch"

When the basic quadratic relation is multiplied by a value between 0 and 1, for example with:

$y = 0.5x^2$ … then the graph is compressed* .

* formally: "scale factor of 0.5 creating a compression"

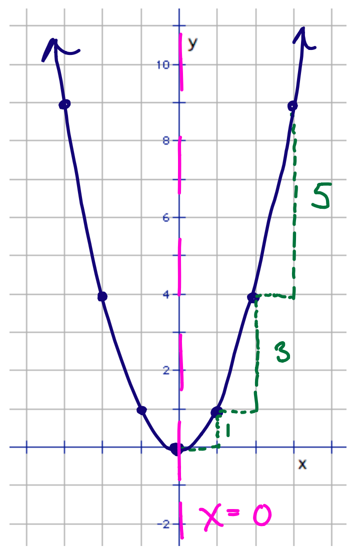
When the basic quadratic relation is multiplied by a negative value, for example with:

$y = -x^2$ … then the graph is reflected (opens downward)

By convention, this value is referred to as a, as in: $y = ax^2$

Example 1

a) Graph the relation $y = x^2$.



i) Direction of opening?
up

ii) Vertex co-ordinates?
(0, 0)

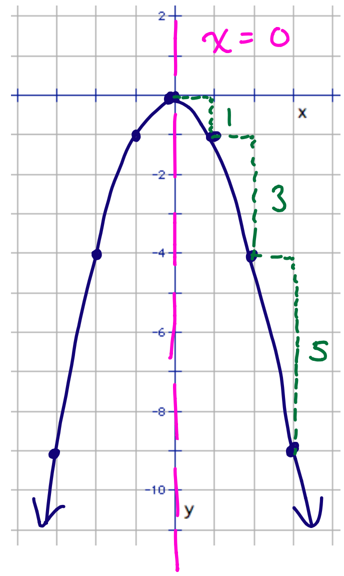
iii) Eq'n of axis of symmetry?
$x = 0$

iv) Max/min value is?
max is ∞ min is 0

v) Values x may take?
$-\infty$ to $+\infty$

vi) Values y may take?
0 to $+\infty$

b) Graph the relation $y = -x^2$



i) Direction of opening?
down

ii) Vertex co-ordinates?
(0, 0)

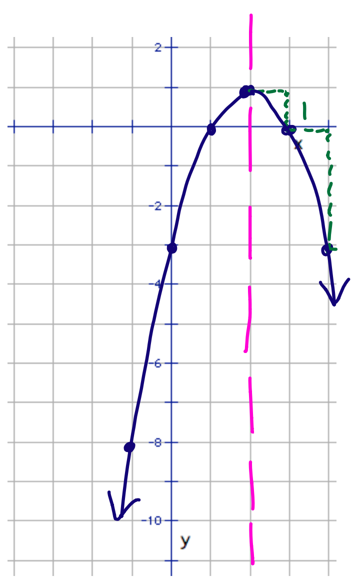
iii) Eq'n of axis of symmetry?
$x = 0$

iv) Max/min value is?
max is 0 min is $-\infty$

v) Values x may take?
$-\infty$ to $+\infty$

vi) Values y may take?
$-\infty$ to 0

c) Graph the relation $y = -(x-2)^2 + 1$



i) Direction of opening?
down

ii) Vertex co-ordinates?
(2, 1)

iii) Eq'n of axis of symmetry?
$x = 2$

iv) Max/min value is?
max is 1 min is $-\infty$

v) Values x may take?
$-\infty$ to $+\infty$

vi) Values y may take?
$-\infty$ to 1

Example 2

$y = 2(x-0)^2 - 5$
a h k

Graph the quadratic function $y = 2x^2 - 5$.

We still start by graphing the vertex.

Where is the vertex? $(0, -5)$.
h k

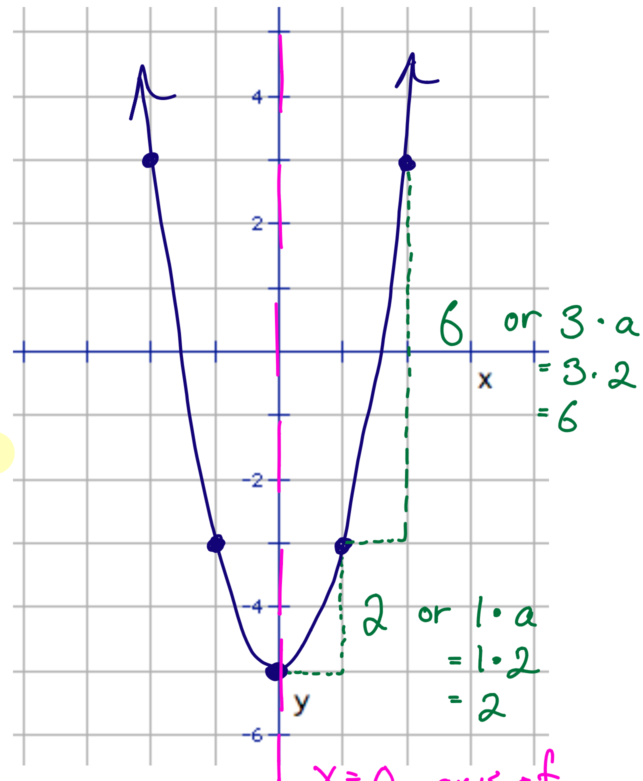
☆ important consolidation

When there is an a value in the quadratic function, multiply the step-pattern by a.

So instead of, from the vertex, (1, 3, 5, …) we will use (2, 6, 10…) to plot the points on the right side.

Then reflect the points in the y-axis to get the “other half”.

Again, note that our plotted points are still correct, when compared to the table of values:



x=0 axis of symmetry

x	y
-3	13
-2	3
-1	-3
0	-5
1	-3
2	3
3	13

… except that plotting with the “step pattern” is much faster than directly computing the table of values.

a) What is the direction of opening? up

b) What are the co-ordinates of the vertex? $(0, -5)$

c) What is the equation of the axis of symmetry? $x = 0$

d) Maximum/minimum value is? max is ∞ min is -5

e) Values x may take? $-\infty$ to $+\infty$

f) Values y may take? -5 to $+\infty$

Opportunity to Learn (Check website for PDF when discussion is over).

All visible questions (including C1, C2, and C3) on the next page. We will take these up in our next class. You may use Desmos to check your work where helpful.

Communicate Your Understanding

C1 Why is the vertical line through the vertex called the axis of symmetry? Illustrate with an example.

C2 When describing the transformation from $y = x^2$ to $y = 2x^2$, you say that it has been stretched vertically by a factor of 2, instead of compressed horizontally. Explain why vertical stretches are used in descriptions.

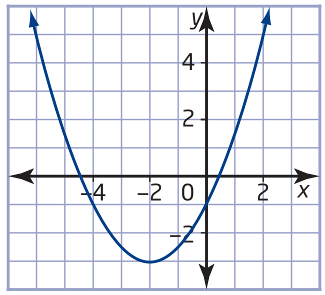
C3 Which equation is correct for the graph shown? Explain your reasoning.

A $y = (x + 2)^2 - 3$

B $y = \frac{1}{3}(x + 2)^2 - 3$

C $y = \frac{1}{2}(x + 2)^2 - 3$

D $y = -2(x + 2)^2 - 3$



Practise

For help with questions 1 and 2, see Example 1.

1. Copy and complete the table for each parabola. Replace the heading for the second column with the equation for the parabola.

Property	$y = a(x - h)^2 + k$
Vertex	
Axis of symmetry	
Stretch or compression factor relative to $y = x^2$	
Direction of opening	
Values x may take	
Values y may take	

a) $y = (x - 4)^2$

b) $y = (x - 2)^2 - 4$

c) $y = (x + 3)^2 - 2$

d) $y = \frac{1}{2}(x + 1)^2 + 5$

e) $y = (x - 7)^2 - 3$

f) $y = -(x - 1)^2 + 7$

g) $y = 2(x - 4)^2 - 5$

h) $y = -3(x + 4)^2 - 2$

2. Sketch each parabola in question 1.

For help with questions 3 to 7, see Example 2.

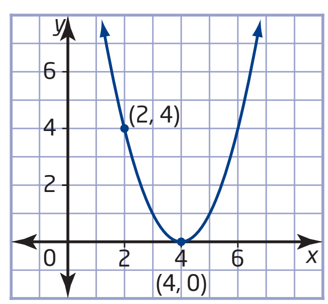
3. Write an equation for the parabola with vertex at (2, 3), opening upward, and with no vertical stretch.

4. Write an equation for the parabola with vertex at (−3, 0), opening downward, and with a vertical stretch of factor 2.

5. Write an equation for the parabola with vertex at (4, −1), opening upward, and with a vertical compression of factor 0.3.

6. Write an equation for each parabola.

a)



b)