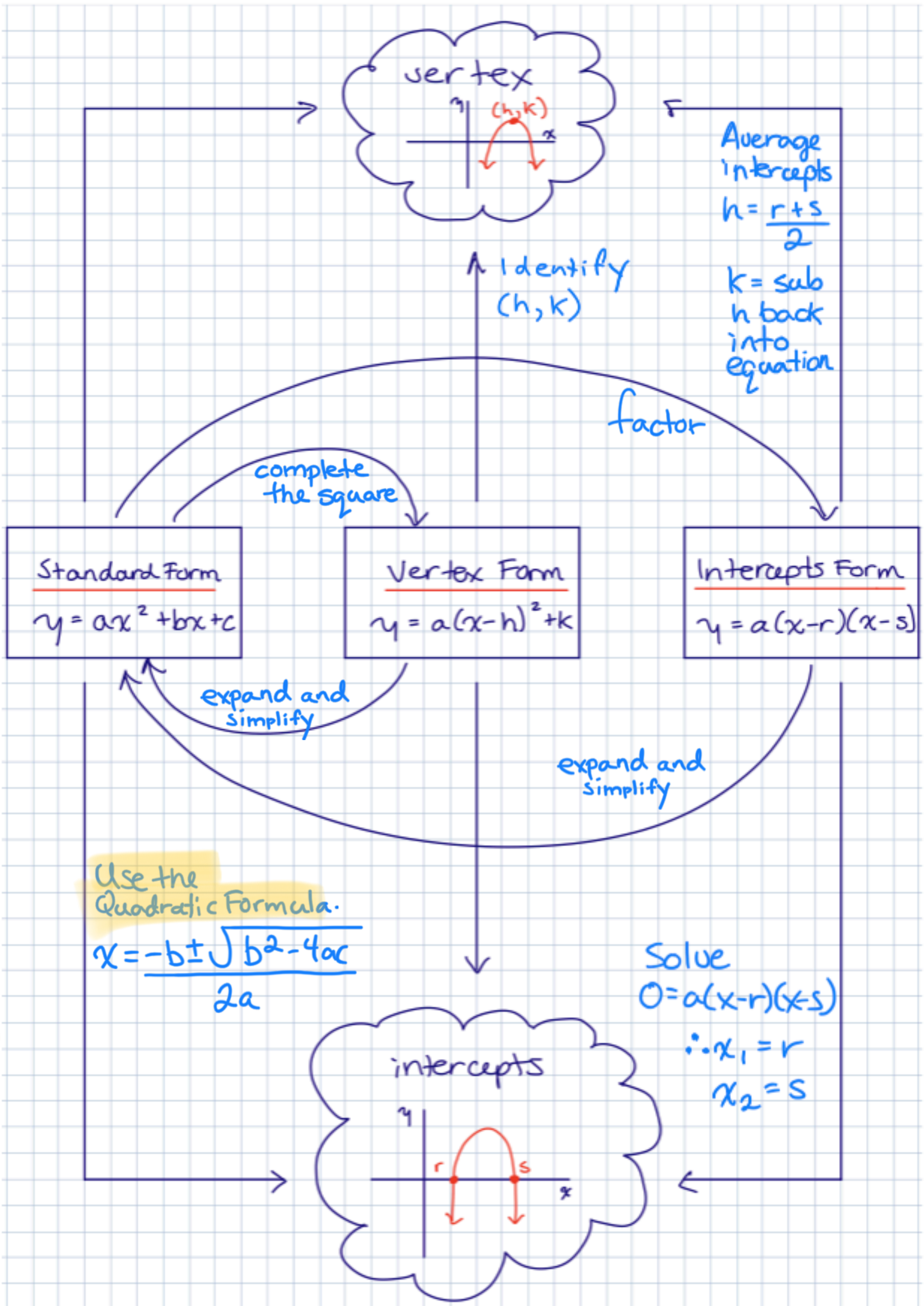
The Quadratic Formula

Quadratic Relations Concept Map

new, from today's lesson



The Quadratic Formula

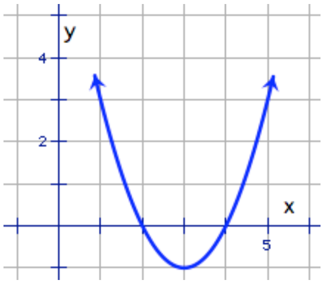
Recall

We already know two ways to solve a quadratic equation.

1. Solve by graphing.

   (look for where the graph crosses the x-axis)

   $\therefore$ solutions are $x=2$ and $x=4$.



2. Solve by factoring.

$$x^2+7x+12=0$$
$$(x+3)(x+4)=0$$

   $\therefore$ solutions are $x=-3$ and $x=-4$.

What happens when these methods don't work well?

For example, what are the solution(s) to the quadratic equation: (a.k.a. what are the roots or x-intercepts?)

$$x^2-3x+1=0$$

1 we could factor... or, could we? (actually cannot, no factors of 1 have a sum of -3)

2 we could complete the square, get vertex, plot points using step pattern, and estimate the x-intercepts visually (but that is time-consuming and would be an estimation... not very precise)

The Quadratic Formula (when all else fails…)

Given a quadratic equation in the form:

$$ax^2+bx+c=0$$

… you can find the solutions / roots / x–intercepts / zeroes using this formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

"Where does this formula come from?"

You can find out here.

Example 1 Solve each quadratic equation.

a) $4x^2 - 12x + 9 = 0$

Remember… $ax^2 + bx + c = 0$ so: $a = 4$, $b = -12$, $c = 9$

To solve, just sub into the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)}$$

$$x = \frac{12 \pm \sqrt{144 - 144}}{8}$$

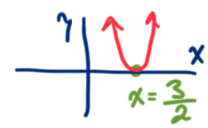
$$x = \frac{12 \pm \sqrt{0}}{8}$$

$$x = \frac{3}{2}$$

$\therefore$ the solution is $x = \frac{3}{2}$

(This quadratic relation touches the x-axis once.)

eg.



Consider: what does this mean? How many times does the graph of this quadratic cross the x-axis? (We'll come back to this later).

b) $x^2 - 2x + 3 = 0$ So… $a = 1$ $b = -2$ $c = 3$

Sub:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 - 12}}{2}$$

$$x = \frac{2 \pm \sqrt{-8}}{2}$$

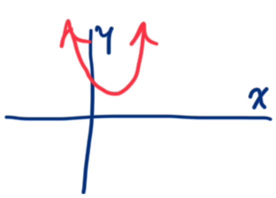
STOP! We cannot take the square root of a negative number.

There are no solutions.

Consider: what does this mean? Where would the graph of this quadratic cross the x-axis?

This quadratic does not cross the x-axis.

eg



c) $x^2 - 3x + 1 = 0$ So… $a = 1$ $b = -3$ $c = 1$

Sub:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9-4}}{2}$$

$$= \frac{3 \pm \sqrt{5}}{2}$$

At this point, we can "split up" the solutions.

$x = \frac{3+\sqrt{5}}{2}$ or $x = \frac{3-\sqrt{5}}{2}$

We get two answers because of the $\pm$

These answers are exact because they are left in radical form (the $\sqrt{\ }$ sign remains)

Sometimes we need <u>approximate</u> answers, so we keep going (rounding off the $\sqrt{5}$).

$x \doteq \frac{3+2.24}{2}$ or $x \doteq \frac{3-2.24}{2}$

$x \doteq \frac{5.24}{2}$ $x \doteq \frac{0.76}{2}$

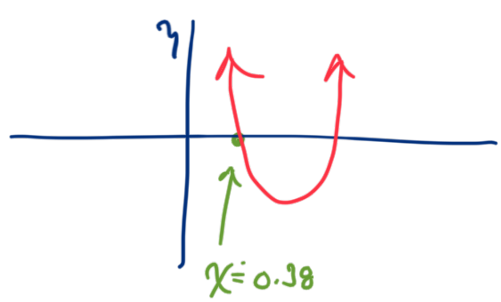
$x \doteq 2.62$ $x \doteq 0.38$

$\therefore$ the solutions are $x = \frac{3+\sqrt{5}}{2}$ or $x = \frac{3-\sqrt{5}}{2}$ (exact)

or $x \doteq 2.62$ or $x \doteq 0.38$ (approximate)

<u>Consider</u>: we have two solutions for x. What does this mean? How many times does this quadratic cross the x-axis?

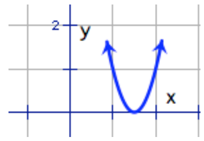
The quadratic crosses the x-axis twice.



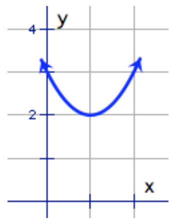
Follow-up: The Discriminant

Look back at parts a), b), and c) in example 1. How many solutions were there in each case?

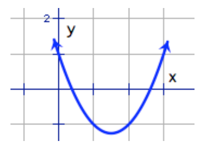
Part a) had one solution, $x = \frac{3}{2}$ Graphically:



Part b) had no solutions. Graphically:



Part c) had two solutions, $x = \frac{3+\sqrt{5}}{2}$ or $x = \frac{3-\sqrt{5}}{2}$ Graphically:



We can predict the number of solutions by examining the discriminant.

What is the discriminant? It's the $b^2 - 4ac$ part of the quadratic formula, in short, everything under the $\sqrt{\ }$ sign.

1. If $b^2 - 4ac = 0$ there is one solution (technically, "two real but equal roots").

   In part a), the discriminant is 0, and sure enough, there was one solution.

2. If $b^2 - 4ac < 0$ there are no solutions (technically, "no real roots").

   In part b), the discriminant is $-$, and there were no solutions.

3. If $b^2 - 4ac > 0$ there are two solutions (technically, "two real roots").

   In part c), the discriminant is $+$, and there were two solutions.

That's it. Whew! All of this boils down to: when you can't solve a quadratic equation by graphing or by factoring, you can always break out the quadratic formula and solve it that way.

Opportunity to Learn

Complete all questions in the provided handout that accompanies this lesson.