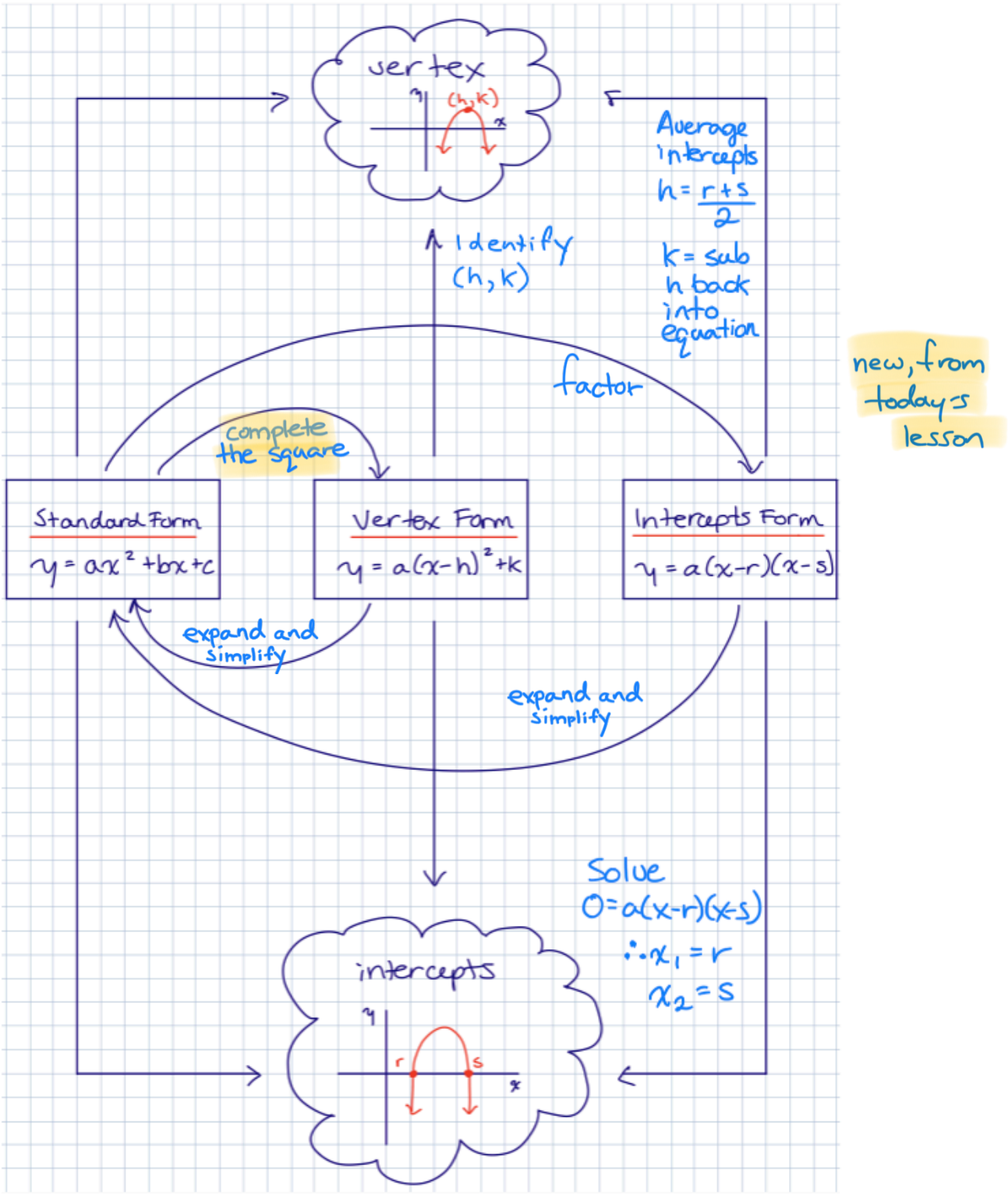
Standard Form to Vertex Form

Quadratic Relations Concept Map



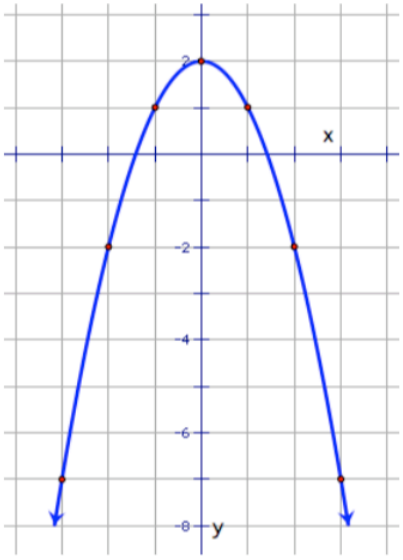
Looking Back

Aside from using words, there are three other representations, or ways, to consider a quadratic relation:

Using symbols (algebra) As a table As a graph

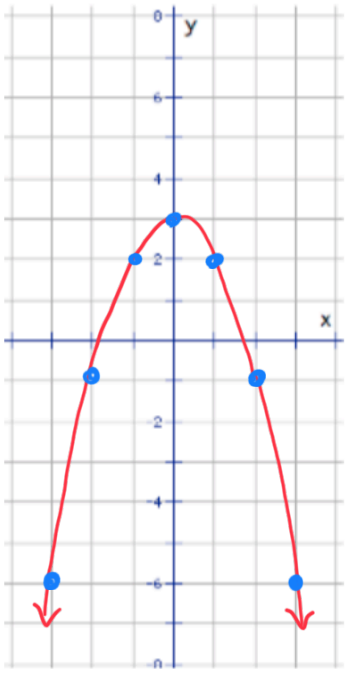
$y = -x^2 + 2$

x	y
-3	-7
-2	-2
-1	1
0	2
1	1
2	-2
3	-7



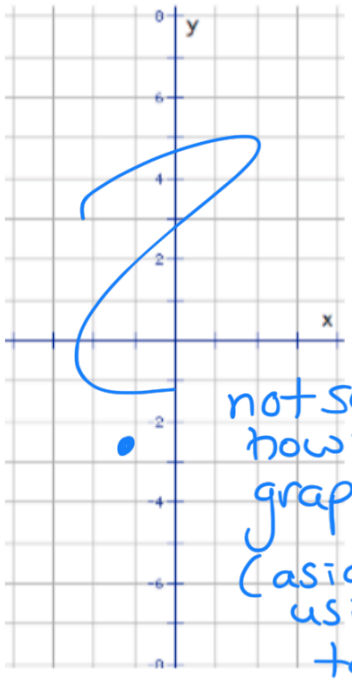
Practice Three quadratic relations are listed below in symbolic form. Write each in graphical form (ie. draw the graph).

$y = -x^2 + 3$



$y = (x-2)^2 - 2$

$y = x^2 + 4x + 5$



not sure how to graph this! (aside from using a table of values... yuck!)

Graphing the third relation may have proven a challenge.

Although it is a quadratic relation (it contains an x^2 term) it is not written in the form:

$y = a(x-h)^2 + k$ … which makes it convenient to graph and analyze.

Completing the Square

We use a method called "completing the square" when given a quadratic relation in the form:

$y = ax^2 + bx + c$ ... to re-write the quadratic relation in the form:

$y = a(x-h)^2 + k$ … which is convenient for graphing and analyzing.

How to Complete the Square

To convert a quadratic relation in the form $y = ax^2 + bx + c$ to the form $y = a(x-h)^2 + k$ … follow these steps:

$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$y = x^2 + 6x + 7$	
$y = x^2 + 6x + 9 - 9 + 7$	Add and subtract the square of half the co-efficient of x
$y = (x^2 + 6x + 9) - 9 + 7$	Group the perfect square trinomial.
$y = (x+3)^2 - 9 + 7$	Factor the perfect square trinomial (write as square of a binomial).
$y = (x+3)^2 - 2$	Simplify outside the brackets.

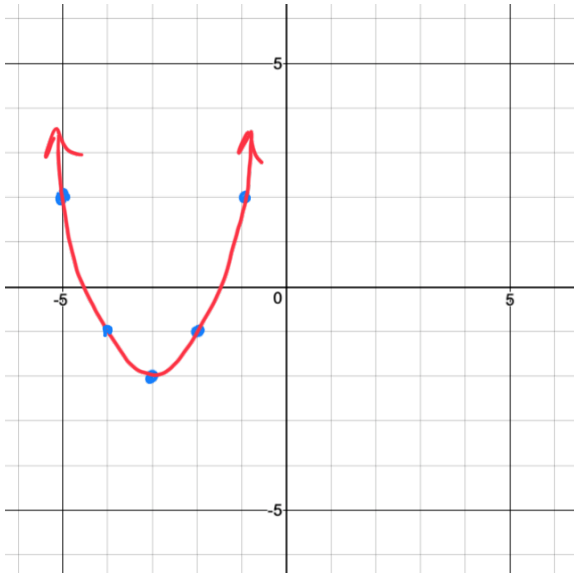
Equivalent Algebraic Expressions

So the expression $y = x^2 + 6x + 7$ can also be written as $y = (x+3)^2 - 2$?

Really?

Are the two relations equivalent?

Let's check using Desmos to graph each relation.



- both expressions produce the same graph
- they are equivalent algebraic expressions

Example 1 Write each relation in the form $y = a(x-h)^2 + k$. State the maximum or minimum point. Then, sketch the graph of each relation.

a) $y = x^2 + 4x + 7$

$\left(\frac{4}{2}\right)^2 = (2)^2 = 4$

$y = x^2 + 4x + 4 - 4 + 7$

$y = (x+2)^2 - 4 + 7$

$y = (x+2)^2 + 3$

minimum point is $(-2, 3)$

b) $y = x^2 - 10x + 24$

$\left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$

$y = x^2 - 10x + 25 - 25 + 24$

$y = (x-5)^2 - 25 + 24$

$y = (x-5)^2 - 1$

minimum point is $(5, -1)$

c) $y = -x^2 - 6x - 8$

Factor out the -1 first!

$y = -(x^2 + 6x + 8)$

$y = -(x^2 + 6x + 9 - 9 + 8)$

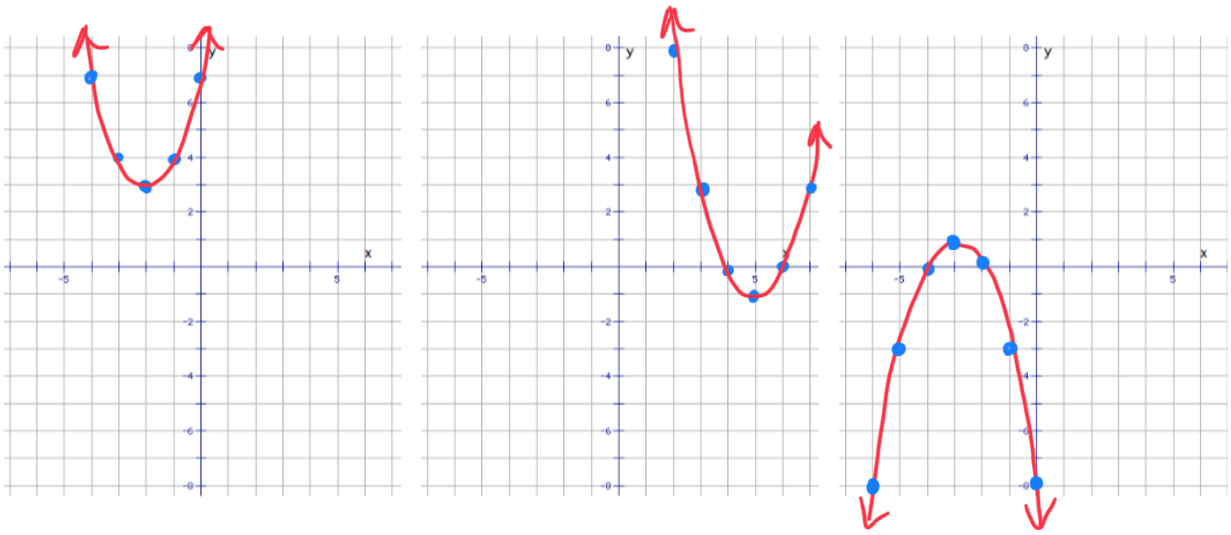
$y = -((x+3)^2 - 9 + 8)$

$y = -((x+3)^2 - 1)$

$y = -(x+3)^2 + 1$

Distribute the -1 again to finish up!

maximum point is $(-3, 1)$



Opportunity to Learn

Complete all questions in the provided handout that accompanies this lesson.