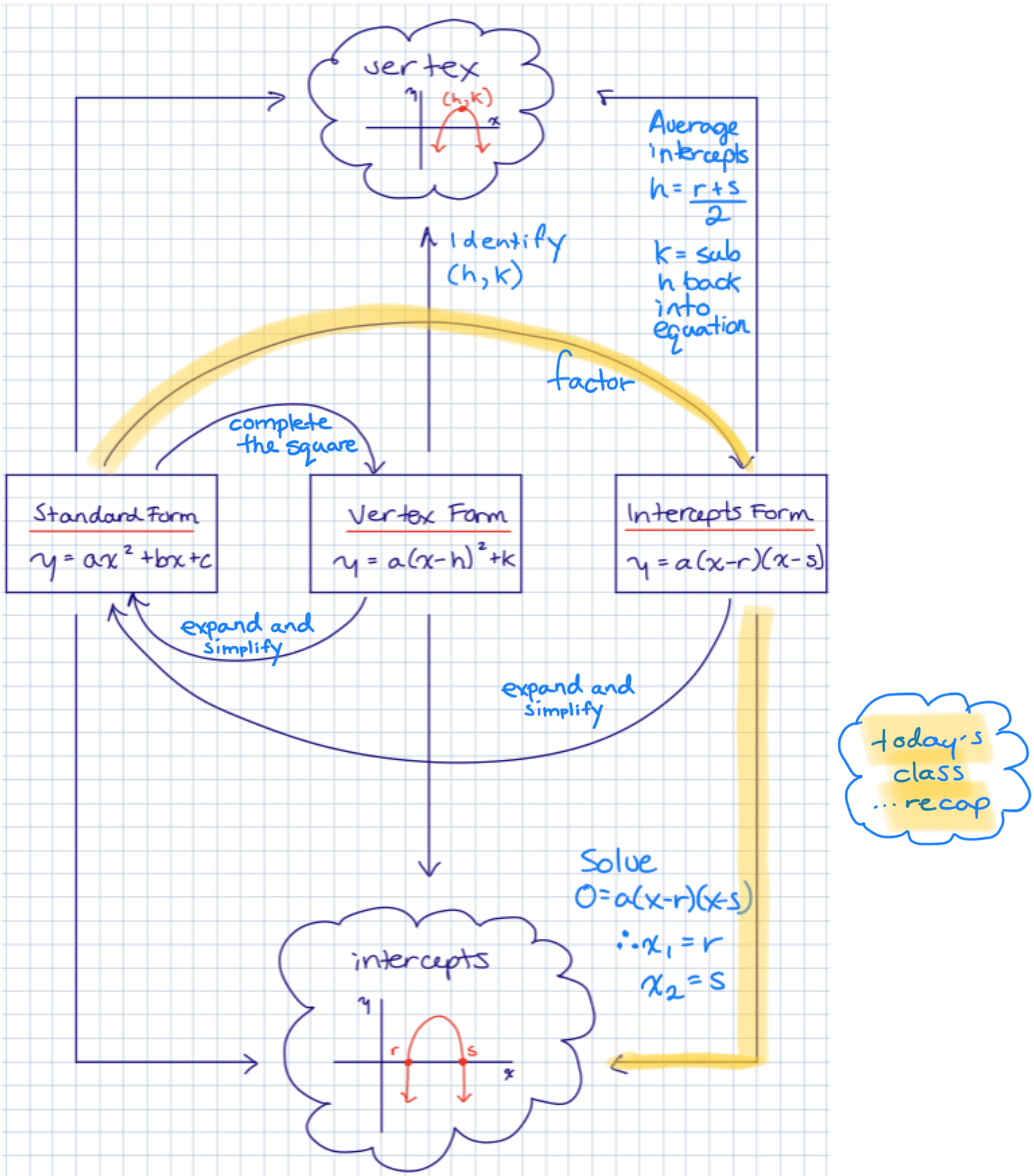
Solving Quadratic Equations by Factoring

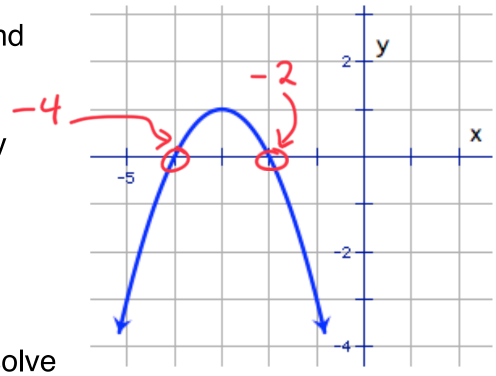
Quadratic Relations Concept Map



Introduction

The flight path of a toy rocket, a ball, or any projectile can be predicted using a quadratic model. This model can also be used to determine when and where a projectile will land (the x-axis is generally used to represent the ground).

In these cases… to find where something lands… we then care about what the x-intercepts are… that is, where a parabola touches the x-axis.



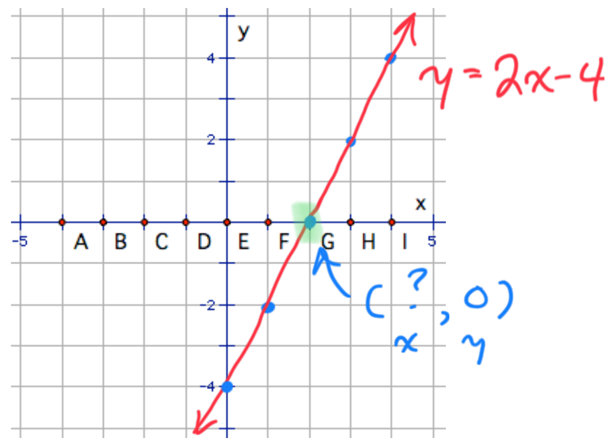
"Solving quadratic equations" just means to find the x-intercepts of the parabola.

You can find the x-intercepts of a parabola by graphing. What are the x-intercepts of the parabola shown at right?

$x=-4$ $x=-2$

Graphing takes time, however… <u>factoring</u> to solve quadratic equations is another method that can be used.

Solving Quadratic Equations



First, what is the y value for any point on the x-axis? That is, an x-intercept has what y value?

Consider points A through I shown to the right.

The y value for any point on the x-axis is: 0

Now, before we look at solving <u>quadratic</u> equations, let's briefly revisit <u>linear</u> equations.

How would you find the x-intercept of the linear equation $y=2x-4$?

$y=2x-4$

Sub $y=0$ and solve: $0=2x-4$

$+4 \quad +4$

$\frac{4}{2}=\frac{2x}{2} \longrightarrow 2=x$

So, the x-intercept of the linear equation is: $x=2$

Solving a quadratic equation is no different.

What are the x-intercepts of $y = x^2 - 6x + 8$? We sub in $y = 0$ and solve:

$0 = x^2 - 6x + 8$ Now at this point, it is hard to see what the value(s) of x are.
$0 = (x-4)(x-2)$ So, we factor the right side.

Now we can see the solutions for x. The x-intercepts are $x = 4$ or $x = 2$.

How did we get this? $x = 4$ is one value that makes the equation $0 = (x-4)(x-2)$ true. For example:

$0 = (x-4)(x-2)$

$LS = 0$

$RS = (x-4)(x-2)$
$= (4-4)(4-2)$
$= (0)(2)$
$= 0$

$x = 2$ is the other value that makes the equation "true"…

$0 = (x-4)(x-2)$

$LS = 0$

$RS = (x-4)(x-2)$
$= (2-4)(2-2)$
$= (-2)(0)$
$= 0$

Example 1

Solve each quadratic equation, by factoring.

a) $0 = x^2 - x - 6$ We factor the right side...

$0 = (x-3)(x+2)$

$x - 3 = 0$
$+3 \quad +3$
$x = 3$

$x + 2 = 0$
$-2 \quad -2$
$x = -2$

what makes the right side also equal zero?

$\therefore$ x-intercepts are
$x = 3$ or $x = -2$

b) $2x^2 + 4x = 0$ We can common factor...

$2x(x+2) = 0$

$\frac{2x}{2} = \frac{0}{2}$
$x = 0$

$x + 2 = 0$
$-2 \quad -2$
$x = -2$

what makes the left side also equal zero?

$\therefore$ x-intercepts are
$x = 0$ or $x = -2$

c) $5x^2 + 19x - 4 = 0$ Complex trinomial...

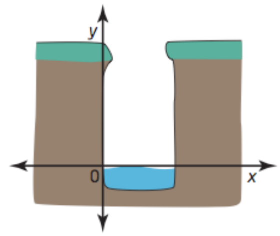
$5x^2 + 20x - x - 4 = 0$ -20
$5x(x+4) - 1(x+4) = 0$ $20, -1$
$(x+4)(5x-1) = 0$

$x + 4 = 0$
$-4 \quad -4$
$x = -4$

$\therefore$ x-intercepts are
$x = -4$ or $x = \frac{1}{5}$.

Example 2

The path of a stone thrown into a ravine is modelled by the quadratic relation $y = -x^2 + 5x + 84$, where x represents the distance, in metres, travelled horizontally and y represents the height, in metres, above the surface of the river at the bottom of the ravine. How far does the stone travel horizontally before it hits the water?



When will the stone hit the water? At the x-axis (when y=0).

So...
$$y = -x^2 + 5x + 84$$
$$\frac{0}{-1} = \frac{-x^2 + 5x + 84}{-1}$$

* Hmm. $-x^2$.
* Since this is an equation... lets divide both sides by -1.

$$0 = x^2 - 5x - 84$$
$$0 = (x+7)(x-12)$$

$x + 7 = 0$ $\quad$ $x - 12 = 0$

$x = -7$ $\quad$ $x = 12$

Now... x represents a horizontal distance.
A negative value, in this context, goes backward... into the cliff.
∴ we discard that intercept.
∴ $x = 12$ ... the stone travels 12 metres horizontally before it hits the water in the ravine.

Opportunity to Learn

Complete all questions in the provided handout that accompanies this lesson.