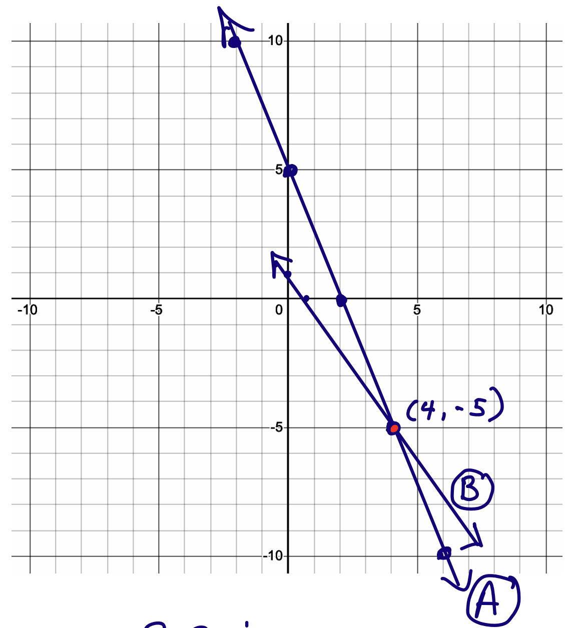
Solving Linear Systems by Elimination

Consider the following linear system:

$$5x + 2y = 10 \quad (A)$$
$$3x + 2y = 2 \quad (B)$$

Solve this linear system by graphing:



By graphing, the solution appears to be: (4 , -5)

Now consider solving the linear system by substitution. Does it look like it will be easy to solve using this method? Make some rough notes. Why or why not?

No, because no matter which variable we isolate we will end up with a fraction for a co-efficient.

e.g.: From (A):

$5x + 2y = 10$

$-5x \quad -5x$

$\frac{2y}{2} = \frac{-5x + 10}{2}$

$y = -\frac{5}{2}x + 5$

There is another method. It is called solving linear systems by elimination. Here's how it works:

Ex.1

$$\begin{array}{r} 5x + 2y = 10 \\ - \quad 3x + 2y = 2 \\ \hline 2x \quad = 8 \end{array}$$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

1. Look for terms that have matching co-efficients.
2. Add or subtract to eliminate a variable.
3. Continue to solve.
4. Substitute value found into either equation.

Sub into (B)

$3x + 2y = 2$

$3(4) + 2y = 2$

$12 + 2y = 2$

$-12 \quad -12$

$\frac{2y}{2} = \frac{-10}{2}$

$y = -5$

$\therefore$, the solution appears to be $(4, -5)$.

Ex. 2 Solve by elimination.

$$\begin{array}{rrcl} & 4x+5y &=& 7 \\ + & -4x-2y &=& -1 \\ \hline & 3y &=& 6 \end{array}$$

$$\frac{3y}{3} = \frac{6}{3}$$

$$y = 2$$

Sub into (A):

$$4x+5y=7$$
$$4x+5(2)=7$$
$$4x+10=7$$
$$-10 \quad -10$$
$$4x=-3$$
$$\frac{4x}{4}=\frac{-3}{4}$$
$$x=-\frac{3}{4}$$

∴ sol'n appears to be $\left(-\frac{3}{4}, 2\right)$.

Fractions are OK!

Ex. 3 Solve by elimination.

$$2x-3y=2 \quad (A)$$
$$5x+6y=5 \quad (B)$$

Hmm... no co-efficients match... but... what if we multiply both sides of (A) by 2?

$$\begin{array}{rrcll} & 4x-6y &=& 4 & (A)_o \times 2 \Rightarrow (A)_n \\ + & 5x+6y &=& 5 & \\ \hline & 9x &=& 9 & \end{array}$$

As long as we multiply both sides by same value, relation remains equivalent.

Now we can add the equations to eliminate y.

$$\frac{9x}{9} = \frac{9}{9}$$

$$x=1$$

Sub into (B):

$$5x+6y=5$$
$$5(1)+6y=5$$
$$5+6y=5$$
$$-5 \quad -5$$
$$6y=0$$
$$\frac{6y}{6}=\frac{0}{6}$$

$$y=0$$

∴ the sol'n appears to be $(1, 0)$.

Ex. 4 Solve by elimination.

$3x - 7y = 13$ (A)
$4x - 5y = 13$ (B)

If needed to get matching co-efficients, we can manipulate **both** equations!

$12x - 28y = 52$ (A) $\times 4 \Rightarrow A_n$
$12x - 15y = 39$ (B) $\times 3 \Rightarrow B_n$

Now we can subtract to eliminate x.

$(-)$

$$\frac{-13y}{-13} = \frac{13}{-13}$$

$y = -1$

Sub into (A):

$3x - 7y = 13$
$3x - 7(-1) = 13$
$3x + 7 = 13$
$\quad -7 \quad -7$

$$\frac{3x}{3} = \frac{6}{3}$$

$x = 2$

$\therefore$, the sol'n appears to be $(2, -1)$.