

## Solving and Analyzing Linear Systems of Equations

### Solving Linear Systems

What is a linear system? It is simply two equations of lines. For example:

$$y = 4x + 3$$

$$y = -2x + 1$$

What does it mean to solve a linear system of equations? It just means to find where the graphs of the equations cross. That is, to find the co-ordinates of the point where the two lines meet.

#### Example 1

Solve the given linear system by graphing.

$$y = x + 1$$

$$2x - y = 2$$

First, we graph each equation.

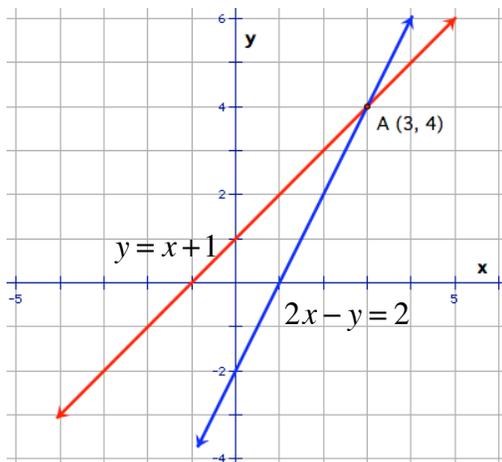
The first equation is in  $y = mx + b$  form. So, graph by plotting the y-intercept,  $b$ .

Then use the slope,  $m$ , and  $\frac{\text{rise}}{\text{run}}$  to plot a few more points. Then draw the line.

The second equation has terms with both  $x$  and  $y$  on the left side.

So, graph using the  $x$  and  $y$  intercepts. Use the “cover up” shortcut we discussed last week: complete the mini-table, plot the intercepts, and draw the line.

$x$	$y$
0	-2
1	0



Now, look at the graph and find where the lines meet.

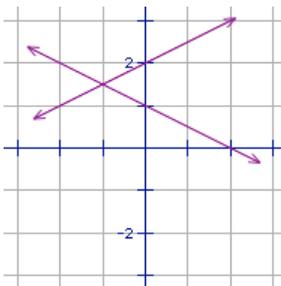
The lines meet at  $(3, 4)$ .

$\therefore$  the solution to the linear system is  $(3, 4)$ .

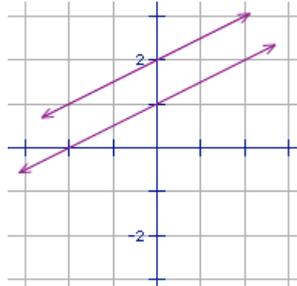
### Analyzing Linear Systems

There are three ways that two lines can meet, as shown below.

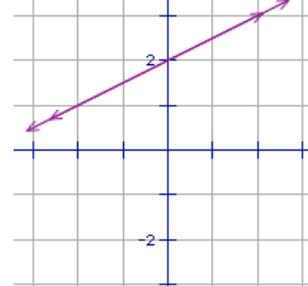
You can identify each case by analyzing the slope and y-intercept of each line.



**intersection**  
 slope: different  
 y-int: different  
  
*one sol'n*  
*(lines cross at one point)*



**parallel**  
 slope: same  
 y-int: different  
  
*no sol'ns*  
*(lines do not cross)*



**parallel, coincident**  
 slope: same  
 y-int: same  
  
*infinite sol'n's*  
*(lines cross at infinitely many points)*

**Example 1** Without graphing, state whether the system of equations has one solution, no solutions, or infinitely many solutions.

a)  $y = -2x + 3$   
 $y = -2x + 1$

slope same  
 y-int different  
 $\therefore$  no sol'n's

b)  $-x + 2y = -6$   
 $2x + y = -1$

solve eqn's for y  
 so that you can  
 analyze

$$\begin{aligned} -x + 2y &= -6 \\ 2y &= x - 6 \\ y &= \frac{1}{2}x - \frac{6}{2} \\ y &= \frac{1}{2}x - 3 \end{aligned}$$

$$\begin{aligned} 2x + y &= -1 \\ y &= -2x - 1 \end{aligned}$$

slope different  
*(now, stop! as soon as you see slope is different, you know there is just one sol'n)*  
 $\therefore$  one sol'n