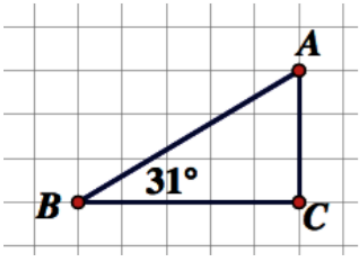
Slope-Angle Right Triangle Conjecture

Warmup

In grade 9, you learned about slope.

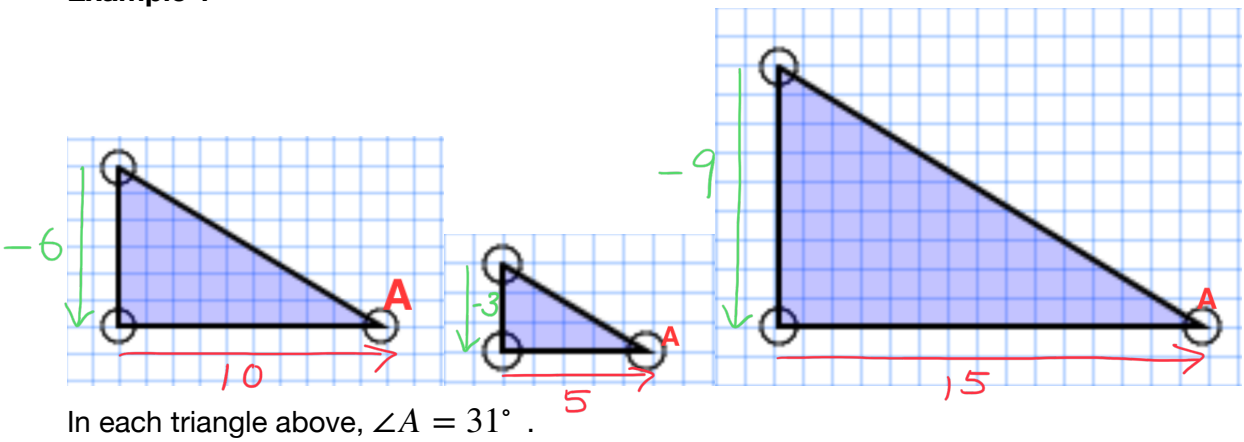
For example, consider the following triangle.



How do you determine the slope of side AB? $\frac{\text{rise}}{\text{run}} = \frac{CA}{BC}$

What is the slope of side AB? $\frac{3}{5}$ or 0.6

Example 1



In each triangle above, $\angle A = 31°$.

Use the diagrams above to complete this table:

Angle	Rise	Run	Slope (to three decimal places)
31°	-6	10	$-\frac{6}{10} = -0.600$
31°	-3	5	$-\frac{3}{5} = -0.600$
31°	-9	15	$-\frac{9}{15} = -0.600$

Then answer these questions:

a) Are the three triangles above similar? yes

b) How do you know the triangles are similar?
- each has right angle (90°)
- each has $\angle A = 31°$
- final angle must be 59° (by $\angle$ATT)

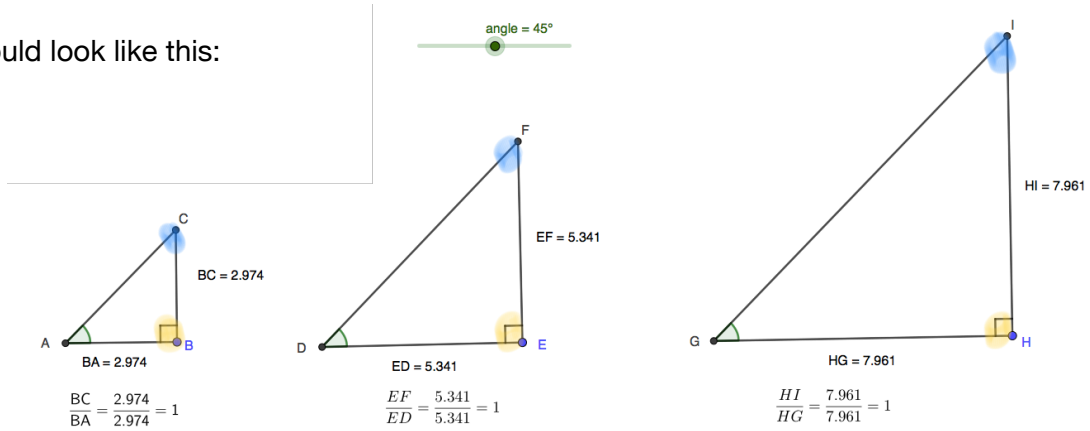
c) What do you notice about the slope of the triangles?

the slope of these triangles is identical.

Example 2

Open this sketch in Geogebra (https://www.geogebra.org/classic/ju3f2kbg).

It should look like this:



Take the time to experiment, based on the instructions in the file.

Then, answer the questions below.

1. Are these triangles congruent? Are they similar? How do you know?

They are not congruent, since side lengths vary. They are similar, since all three corresponding angles are equal (90° angle, the green angle, remaining angle is same by SATT.)

2. Be sure that you adjust the size of each triangle by dragging points B, E, or H. Does that change the slope of the triangle?

No, changing the size (while the angle stays the same) does not change the slope.

3. Change the angle slider. What do you notice about the triangles? What do you notice about the slope?

Changing the slider adjusts the marked angle, changing the shape of each triangle... but I notice all the triangles have the same slope.

4. Make a conjecture (an educated guess) about how the marked (green) angle of these triangles, their slope, and the size of a triangle are all related.

Try making a conjecture by completing this sentence:

Given a certain angle, say, 47°... the three triangles of different sizes will always have the same slope. So... the shape (based on the angle) is the same, the size of the triangles varies, but the slope always matches.

Opportunity to Learn

1. Where is the hypotenuse in these triangles? Label that side with the letter "H".

   Draw an arrow from the marked angle to the side across the triangle. Label this "O" for opposite.

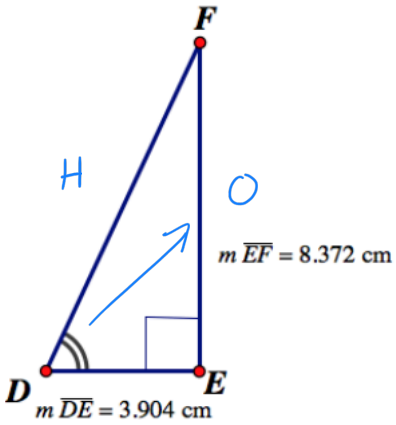
   Label the remaining side "A" for adjacent.

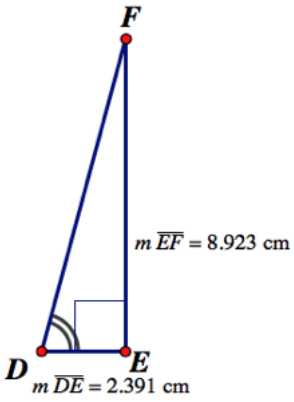
   Calculate the slope for each triangle (round your answer to three decimal places).

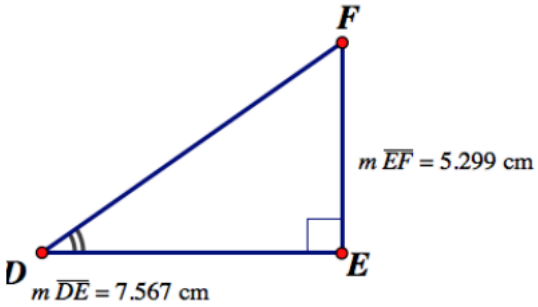
   Take the slope you calculated for each triangle. Locate that slope in this table (http://tinyurl.com/y2z9gef7).

   Finally, grab a protractor. Measure the angle in each triangle.

   What do you notice?







A

slope = $\frac{\text{rise}}{\text{run}}$

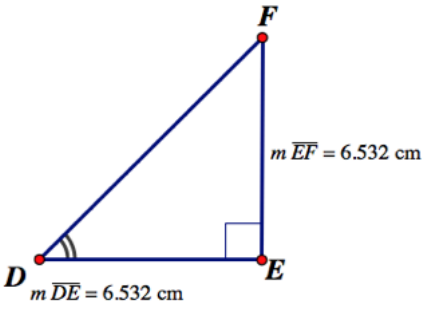
$= \frac{8.372}{3.904}$

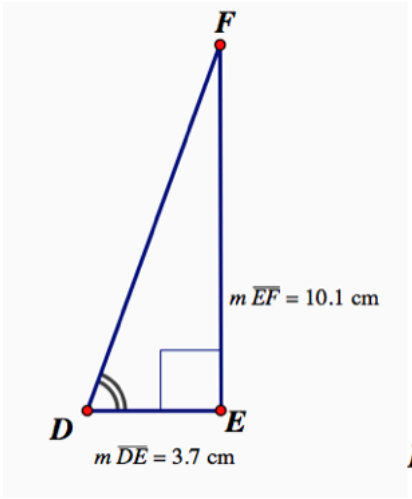
$\div 2.144$

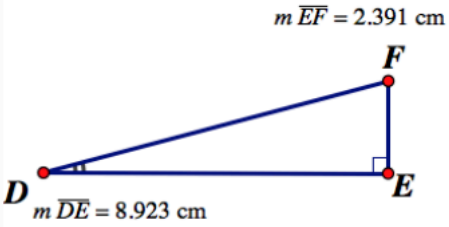
$\angle D = 65°$

(by measuring)

1. (continued)







2. How do you identify the opposite and adjacent sides of a right triangle?

   Are these two sides always the same in a given triangle?

   Why or why not?