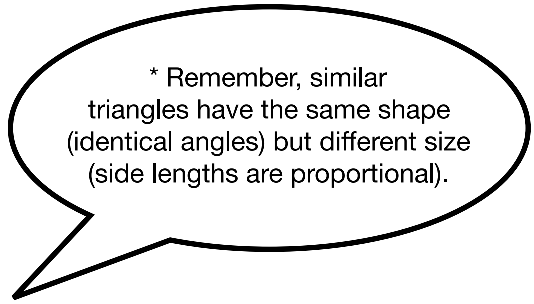
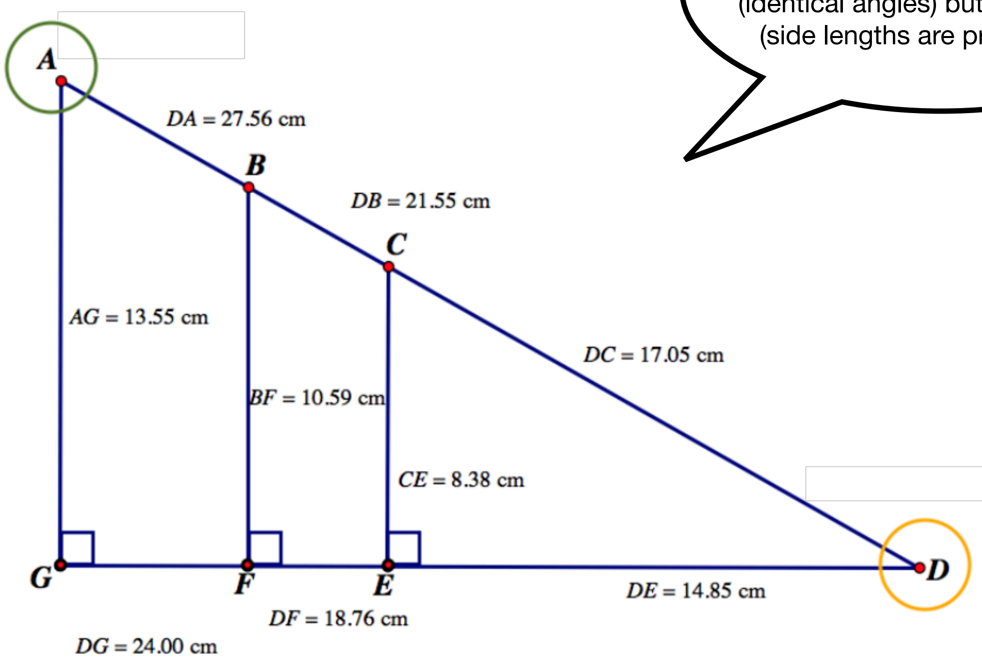
Sine and Cosine Ratios



Example 1



Triangle	Opposite Length Looking From… $\angle D$	Hypotenuse Length	Sine ratio opposite ÷ hypotenuse
CED	8.38	17.05	0.491
BFD	10.59	21.55	0.491
AGD	13.55	27.56	0.491

What do you notice about the ratio of *opposite* ÷ *hypotenuse* for these similar* triangles?

the ratios are the same across the three triangles

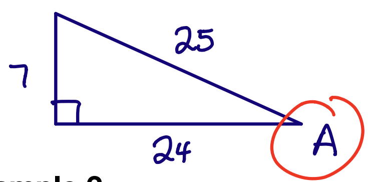
Triangle	Adjacent Length Looking From… $\angle D$	Hypotenuse Length	Cosine ratio adjacent ÷ hypotenuse
CED	14.85	17.05	0.871
BFD	18.76	21.55	0.871
AGD	24.00	27.56	0.871

What do you notice about the ratio of *adjacent* ÷ *hypotenuse* for these similar* triangles?

again, the ratios are the same across the three triangles

Summary

Given a triangle:



$\sin A = \frac{opp}{hyp} = \frac{7}{24}$ $\quad \cos A = \frac{adj}{hyp} = \frac{24}{25}$ $\quad \tan A = \frac{opp}{adj} = \frac{7}{24}$

Example 2

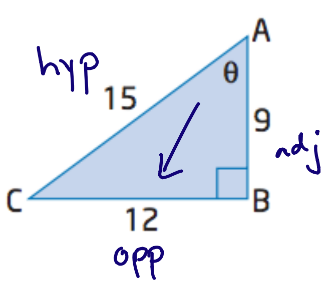
For each triangle shown below:

- identify the sine ratio from the angle marked θ
  - (for short, we'd say "find sin θ")
- identify the cosine ratio from the angle marked θ
  - (for short, we'd say "find cos θ")
- identify the tan ratio from the angle marked θ
  - (for short, we'd say "find tan θ")

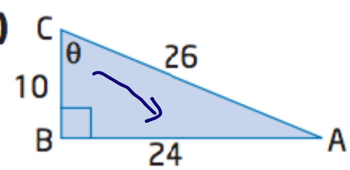
S
O
H
C
A
H
T
O
A

Sometimes helpful to remember the three ratios

a)



b)



$\sin \theta = \frac{opp}{hyp} = \frac{12}{15}$

$\cos \theta = \frac{adj}{hyp} = \frac{9}{15}$

$\tan \theta = \frac{12}{9}$

$\sin \theta = \frac{24}{26}$

$\cos \theta = \frac{10}{26}$

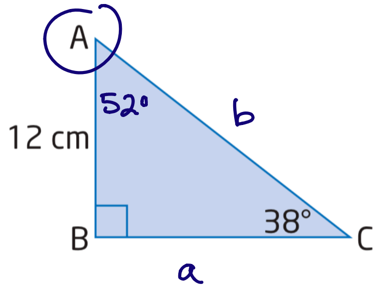
$\tan \theta = \frac{24}{10}$

Example 3

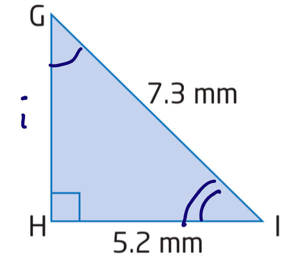
Solve each triangle (find the measure of all angles and side lengths).

(numerous ways to solve a Δ, answers may vary)

a)



b)



$\angle A = 180° - 90° - 38°$
$= 52°$

$\tan 52° = \frac{opp}{adj}$

$\tan 52° = \frac{a}{12}$

$12\left[\tan 52°\right] = \left[\frac{a}{\cancel{12}}\right]\cancel{12}$

$15.4 \doteq a$

$\cos 52° = \frac{adj}{hyp}$

$\cos 52° = \frac{12}{b}$

$b\left[\cos 52°\right] = \left[\frac{12}{\cancel{b}}\right]\cancel{b}$

$\frac{b(\cancel{\cos 52°})}{\cancel{\cos 52°}} = \frac{12}{\cos 52°}$

$b \doteq 19.5$

$hyp^2 = leg_1^2 + leg_2^2$

$7.3^2 = 5.2^2 + i^2$
$-5.2^2 \quad -5.2^2$

$26.25 = i^2$

$\sqrt{26.25} = \sqrt{i^2}$

$5.1 \doteq i$

$\sin G = \frac{opp}{hyp}$

$\sin G = \frac{5.2}{7.3}$

$\sin G \doteq 0.712$

$\angle G = \sin^{-1}(0.712)$

$\angle G \doteq 45.4$

$\therefore \angle I = 180 - 90 - 45.4$
$= 44.6°$

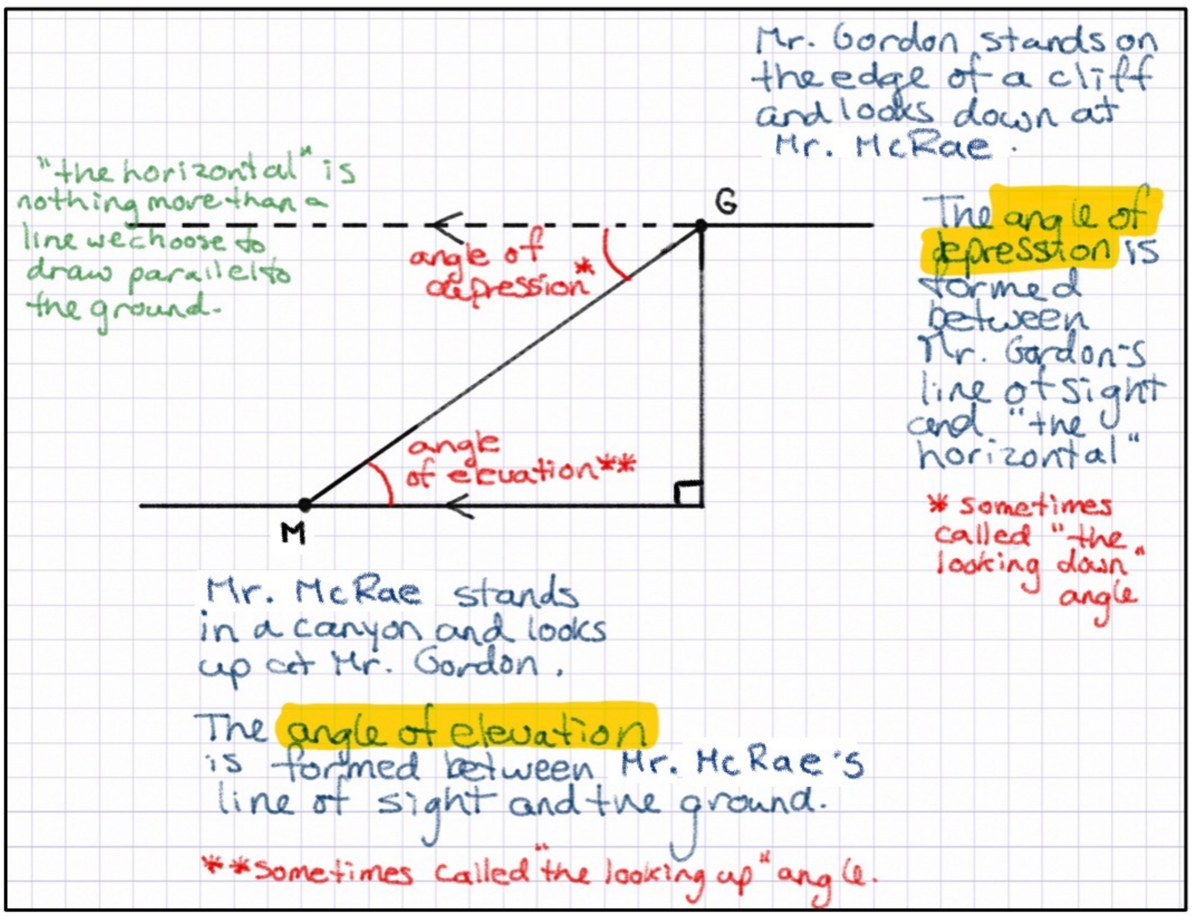
Terminology

NOTE

In some trigonometry questions, you will encounter the terms *angle of elevation* and *angle of depression*. Consider the example at right for an explanation.

Whenever you encounter the terms *angle of elevation* and *angle of depression* the same diagram shape holds true.

Also note that since “the horizontal” (the dotted line) is parallel to the ground, it means that the two angles are equal (by “Z” pattern or alternate angles).

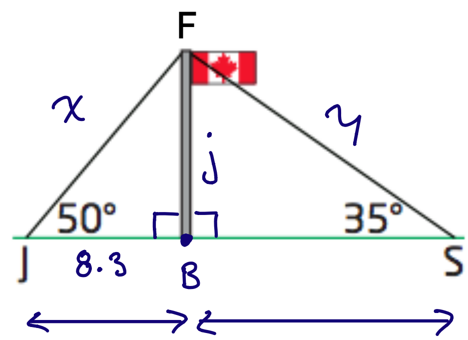


Opportunity to Learn

1. Jack and Sangita are facing each other on opposite sides of a flagpole.

   Two guy wires (wires that support the flagpole) extend from Jack and Sangita's positions, respectively, to the top of the flagpole.

   If Jack is standing 8.3 metres from the base of the flagpole, how long is each guy wire?



Assumptions:

* Ground is flat.
* Flagpole is $\perp$ to ground.

Let j be the height of the flagpole, in metres.
Let x " " length " " left guy wire, in metres.
Let y " " " " " right guy wire, in metres.

$\cos 50° = \dfrac{adj}{hyp}$

$\cos 50 = \dfrac{8.3}{x}$

$x \cdot \cos 50° = 8.3$

$x = \dfrac{8.3}{\cos 50°}$

$x \doteq 12.9$

$\tan 50° = \dfrac{opp}{adj}$

$\tan 50° = \dfrac{j}{8.3}$

$8.3 \cdot \tan 50° = j$

$9.89 \doteq j$

$\sin 35° = \dfrac{opp}{hyp}$

$\sin 35° = \dfrac{9.89}{y}$

$y \cdot \sin 35° = 9.89$

$y = \dfrac{9.89}{\sin 35°}$

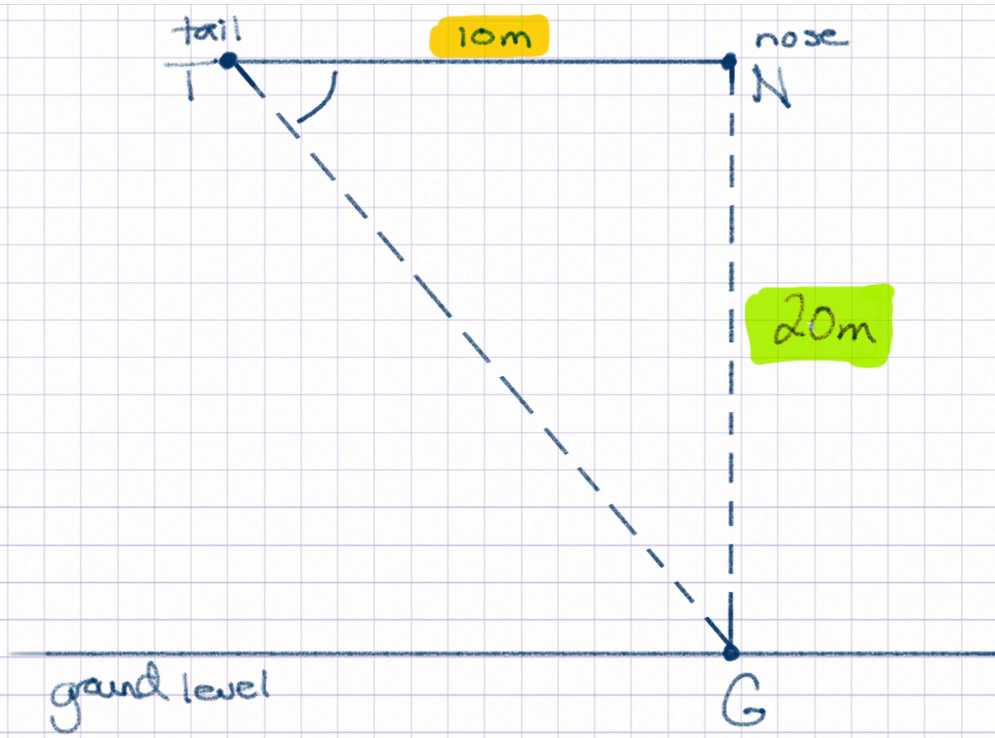
$y \doteq 17.2$

$\therefore$, the left guy wire is about 12.9m long and the right guy wire is " 17.2m long.

2. A special type of aircraft is designed to fly at the very low height of 20 m. To measure such a small altitude, two spotlights are mounted on the aircraft:

- one on the nose, pointing straight down
- another mounted on the tail of the aircraft, 10 m away

Find the angle at which the second light needs to be set, with respect to the body of the aircraft, so that the beams will meet 20 m below the aircraft.



$\tan T = \frac{opp}{adj}$

$= \frac{20}{10}$ $\rightarrow$ $\tan T = 2$

$\therefore \angle T \doteq 63°$

we chose to use the tangent ratio because from the perspective of $\angle T$, we know the lengths of the opposite and adjacent sides

(via lookup in tangent ratio table)

3. Theresa and Branko are competing in a series of outdoor challenges that will eventually lead them to a hidden treasure. Each clue they find helps them find a new clue. Theresa is getting ready to climb a steep cliff to find their next clue at the Lookout Point. She has two options:

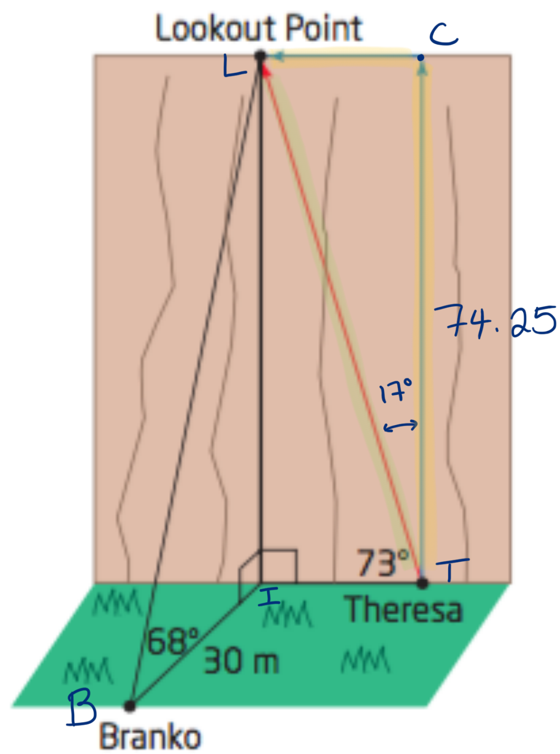
- Option A: Climb straight up the cliff, and then jog over to Lookout Point.
- Option B: Climb directly to Lookout Point along the diagonal shown.

She is awaiting instructions from Branko, who is positioned directly facing Lookout Point at a distance of 30 m from the base of the cliff.

From Branko's point of view, Lookout Point is at an angle of elevation of $68°$. He also observes that the diagonal path up the cliff makes a $73°$ angle with the ground. Branko knows that Theresa can climb at a speed of 1.0 m/s and jog at a speed of 5.0 m/s after a climb.

It is a tight race and seconds count.

Which option should Branko tell Theresa to take: A or B?



Option A $= \overline{CT} + \overline{CL}$

Now $LI \cong CT$ congruent to

$\tan B = \dfrac{LI}{BI}$

$30[\tan 68°] = \left[\dfrac{LI}{30}\right]30$

$74.25 \doteq LI$

$\therefore CT \doteq 74.25$

Assume $CT \perp IT$

Then $\angle CTL = 90° - 73°$
$= 17°$

Now: $\tan CTL = \dfrac{CL}{CT}$

$74.25[\tan 17°] = \left[\dfrac{CL}{74.25}\right]74.25$

$22.7 \doteq CL$

So total time:

☆ $\dfrac{CT}{1} + \dfrac{CL}{5} = \dfrac{74.25}{1} + \dfrac{22.7}{5}$
$\doteq 79$ seconds

☆ time $= \dfrac{\text{distance}}{\text{speed}}$

Option B $= LT$

$\cos LTC = \dfrac{CT}{LT}$

$LT[\cos 17°] = \left[\dfrac{74.25}{LT}\right]LT$

$\dfrac{LT(\cos 17°)}{\cos 17°} = \dfrac{74.25}{\cos 17°}$

$LT = \dfrac{74.25}{\cos 17°}$

$LT \doteq 77.64$

So total time: ☆

$\dfrac{LT}{1} = \dfrac{77.64}{1} = 77.64$ seconds

$\therefore$ Theresa should choose option B as it is faster.