Similar and Congruent Figures

Warmup

Solve each proportion.

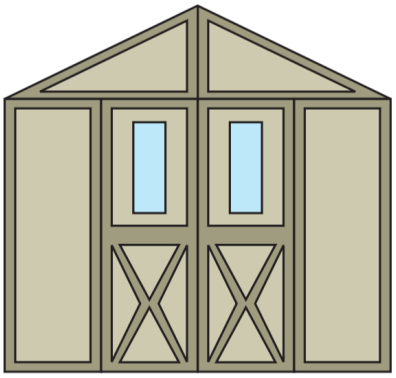
a) $\frac{x}{6} = \frac{1}{2}$

b) $\frac{3}{y} = \frac{2}{3}$

c) $\frac{a}{2} = \frac{2}{a}$

d) $\frac{x+2}{2} = \frac{4}{x}$

Example 1



How can we recognize similar and congruent figures?

a. Look at the front face of the shed.

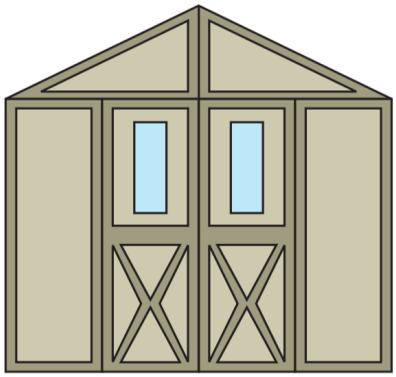
   i. What pairs of congruent figures can you find?
   ii. Copy two or three pairs of congruent figures from the drawing into the space below.

   iii. For each pair of congruent figures you drew, label the vertices. What special properties do the corresponding sides and angles have?

b. Review your work from part a).

   Now, summarize: what are the properties of congruent figures?

c. Look again at the front face of the shed.



i. What pairs of similar, but not congruent, figures can you find?
ii. Copy a pair of similar figures from the drawing into the space below.
iii. Label the vertices of the figures.
What special properties do the corresponding sides and angles have?

d. Complete the table below to consolidate your thoughts.

i. What is true about the corresponding angles in two congruent figures?
In two similar figures?
ii. What is true about the corresponding side lengths in two congruent figures?
In two similar figures?

	corresponding angles are...	**corresponding side lengths are...**
In congruent figures		
In similar figures		

Example 2

If two triangles are congruent, what does that mean?

In short, there are:

Is it possible to prove the congruence of two triangles without meeting all of these conditions?

Shortforms are sometimes used to describe congruency conditions between triangles. Below:

- S means “side”
- A means “angle”
- H means “hypotenuse”

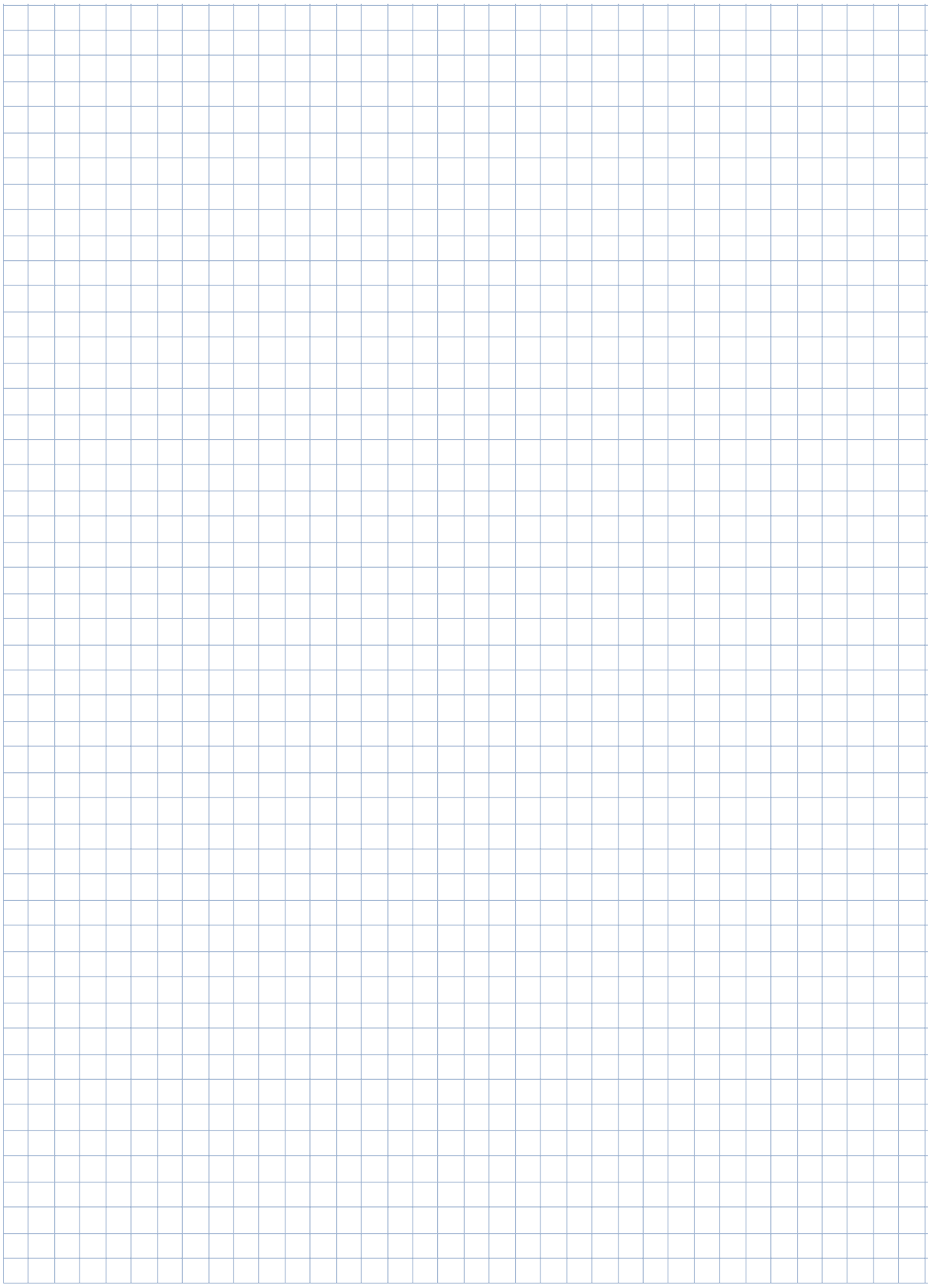
Here are five possible congruency conditions between two triangles:

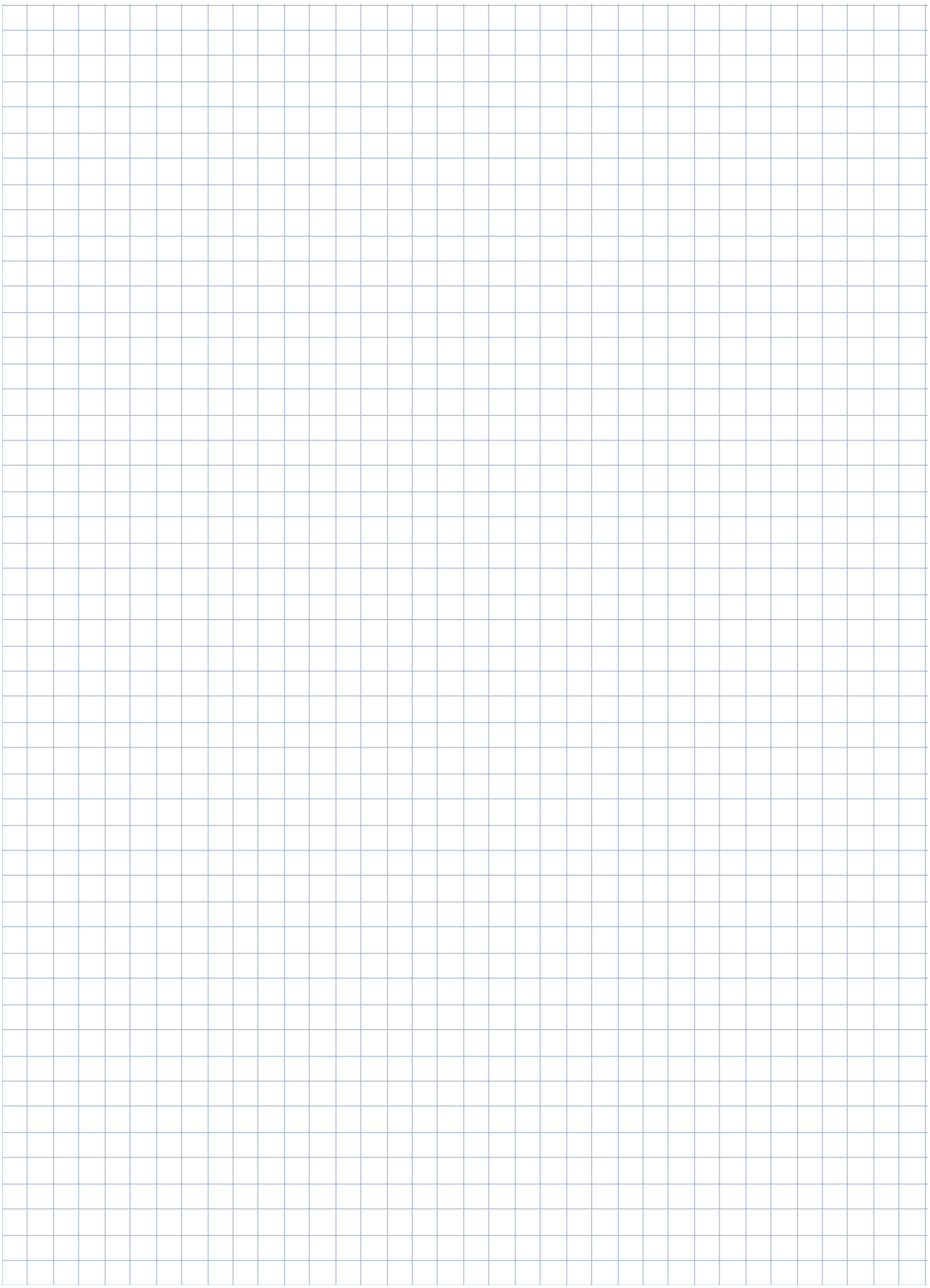
i. SSS ii. SAS iii. ASA iv. AAS v. HS

So the first condition, SSS, suggests that two triangles are congruent if we know only that three corresponding sides are of equal length and know nothing of the angles. Is this true? Why?

The second condition, SAS, suggests that two triangles are congruent if we know only that two corresponding sides are of equal length with one corresponding equal angle “sandwiched between” the equal sides. We know nothing else about the triangles. Can this be true? How?

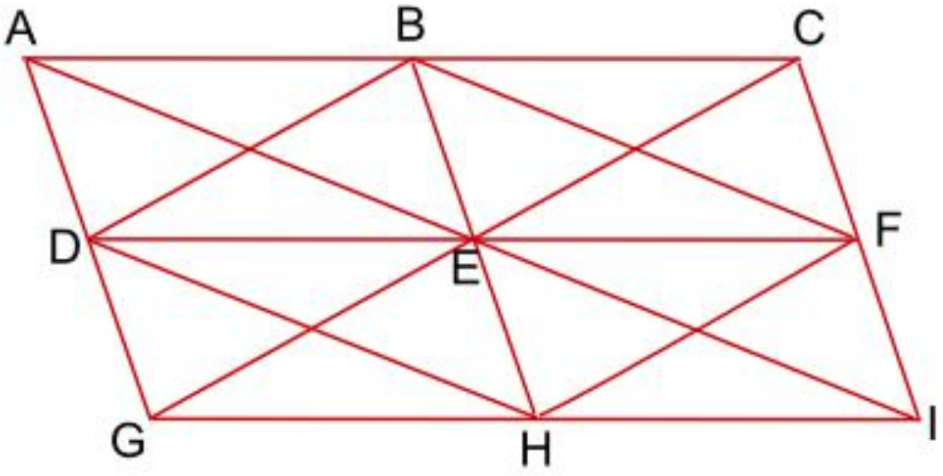
On the following pages, use diagrams and brief written explanations to determine, through your own reasoning, whether all five of these “congruency conditions” are in fact true.



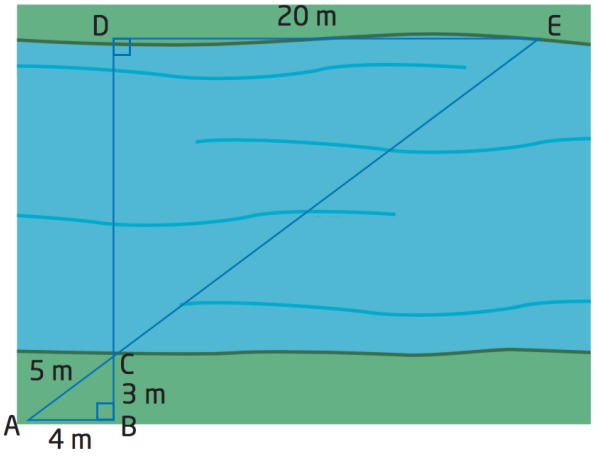


Opportunity to Learn

1. How many triangles congruent to triangle ABE (including itself) are there in the following diagram?



2. Surveyors have staked out the following points on opposite banks of a river.



a) Are triangles ABC and EDC congruent? Are they similar?

b) If the surveyors want to build a footbridge from points C to D, how long will it need to be?

c) If the surveyors want to build a footbridge from points C to E, how long will it need to be?