Thread 4 Review: Similar Triangles and Trigonometry

Solutions to Problems 1 to 15

1. a)

statement	reason
$\angle B = \angle D$	given
$\angle BCA = \angle DCE$	VOA
$\angle A = \angle E$	SAT

$\therefore \triangle BCA \sim \angle DCE$

$\because$ $\triangle$'s are similar (meaning all angles are equal ... and $BC = CD$ and $BA = ED$ ... AC must logically be equal to length of CE. $\therefore \triangle BCA \cong \triangle DCE$

↑ congruent to

b)

statement	reason
$\angle X = \angle Z$	given
$\angle WYX = \angle AYZ$	given
$\angle W = \angle A$	SAT

$\therefore \triangle WYX \sim \triangle AYZ$

2.

$\frac{RP}{HG} = \frac{5}{2.5} = 2$

$\frac{RQ}{HF} = \frac{9}{4.5} = 2$

$\frac{PQ}{GF} = \frac{8}{4} = 2$

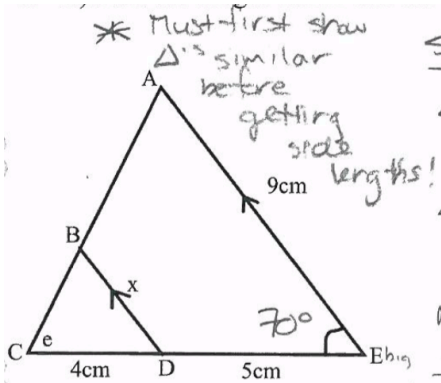
} all ratios of corresponding side lengths are equal

$\therefore \triangle RPQ \sim \triangle HGF$

3.

AE = CE
9 cm
∴ △AEC is isosceles

∴ ∠ACE = $\frac{180-70^\circ}{2}$ = 55° by ITT



* Must first show △'s similar before getting side lengths!

statement	reason
∠BCD = ∠ACE	shared / same angle
∠CDB = ∠CEA	PL-F
∠CBD = ∠CAE	SAT

∴ △BCD ~ △ACE

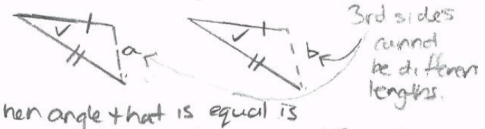
Now...

$\frac{CE}{CD} = \frac{AE}{BD}$ (big/small = big/small)

$\frac{9}{4} = \frac{9}{x}$ → x = 4

4.

Example - Congruent



when angle that is equal is between the equal sides... △'s must be co

Example - Not Congruent

a ≠ b

when the equal angle is not between the equal sides... 3rd sides will not be of equal length.

5. $\angle A = \sin^{-1}\left(\frac{9}{12}\right) = 48.59 = 49^\circ$

$\angle B = \cos^{-1}(0.1841) = 79.39 = 79^\circ$

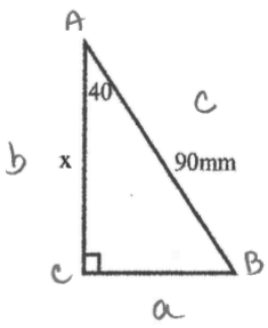
6. $\sin B = \frac{opp}{hyp} = \frac{9}{15} = 0.6$

$\cos B = \frac{adj}{hyp} = \frac{12}{15} = 0.8$

$\tan B = \frac{opp}{adj} = \frac{9}{12} = 0.75$

7.

a.)



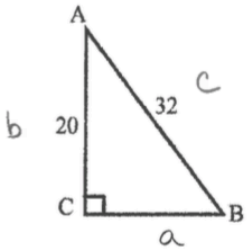
$\angle A = 40^\circ$
$\angle B = 50^\circ$
$\angle C = 90^\circ$
$a = 57.85$
$b = 68.94$
$c = 90$

$\cos 40 = \frac{b}{90} \quad \therefore b = 90\cos 40 = 68.94$

$\angle B = 180 - 40 - 90 = 50^\circ$

$\sin 40 = \frac{a}{90} \quad \therefore a = 90\sin 40 = 57.85$

b.)



$\angle A = 51.32^\circ$
$\angle B = 38.68^\circ$
$\angle C = 90^\circ$
$a = 24.98$
$b = 20$
$c = 32$

$\sin B = \frac{20}{32} \quad \therefore \angle B = \sin^{-1}\left(\frac{20}{32}\right) = 38.68^\circ$

$\angle A = 180 - 90 - 38.68 = 51.32^\circ$

$\tan A = \frac{a}{20}$

$\therefore a = 20\tan(51.32) = 24.98$

8.

the opposite side of $\angle B$ is the adjacent side side of $\angle A$

9.

$\tan 30^\circ = \frac{80}{AC}$

$AC\left[\tan 30^\circ\right] = \left[\frac{80}{AC}\right]AC$

$\frac{AC \cdot \tan 30^\circ}{\tan 30^\circ} = \frac{80}{\tan 30^\circ}$

$AC = \frac{80}{\tan 30^\circ}$

$\tan 45^\circ = \frac{80}{BC}$

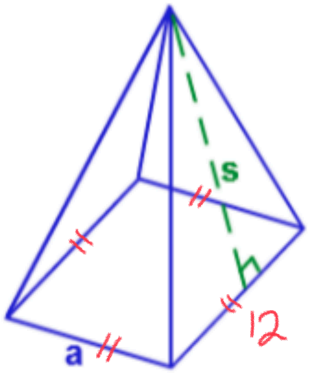
$BC = \frac{80}{\tan 45^\circ}$

Then... $AB = AC - BC$

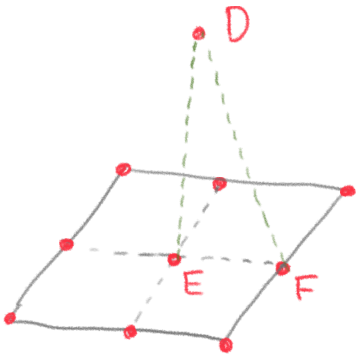
$= \frac{80}{\tan 30^\circ} - \frac{80}{\tan 45^\circ}$

$\doteq 58.6$

10.



Square Pyramid



Let E be the centre-point of the pyramid base (directly below the apex at point D).

Then $\angle DEF = 90°$.

If $a = 12$, then $EF = 6$ when the square base is divided into four equal quadrants.

Now... it is given that DF, the slant height, is 10 units long.

Then...

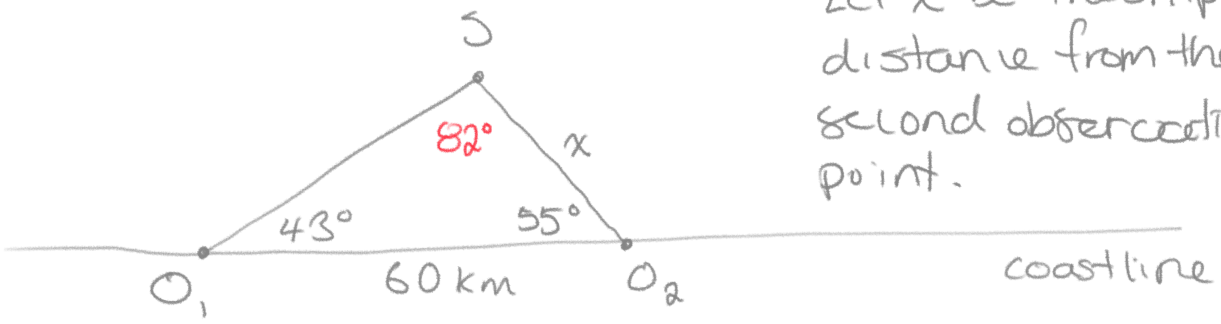
$$\cos DFE = \frac{EF}{DF}$$

$$= \frac{6}{10}$$

$$\angle DFE = \cos^{-1}\left(\frac{6}{10}\right)$$

$$\angle DFE \doteq 53°$$

11.



Let x be the ship's distance from the second observation point.

$\angle S = 82°$ SATT

Then: $\frac{60}{\sin 82°} = \frac{x}{\sin 43°}$

Use Sine Law (angle across from known side)

$\sin 43° \left[\frac{60}{\sin 82°}\right] = \left[\frac{x}{\sin 43°}\right] \sin 43°$

$41 \doteq x$

$\therefore$ the ship is about 41 km from the second observation point.

12.

13.

14.

15.