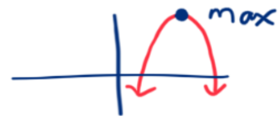
Problem Solving With Quadratics

Recall



Some key vocabulary…

"maximum" / "minimum" / "highest" / "lowest" means… we need the vertex.

"horizontal distance" / "hits the ground" / "intercept", or, we need x, means… we need the x-intercepts (roots or zeroes)

Example 1

y=50

y=0

The shape and size (in metres) of the St. Louis Gateway Arch can be approximately modeled by the equation:

$y = -0.025x^2 + 4.8x - 38.4$

… where x is the horizontal distance in metres from the left side of the arch, and y is the vertical distance in metres above the ground.

a) Find the approximate horizontal distance across the base of the arch, in metres. (need intercepts)

b) Find the approximate horizontal distance across the arch, in metres, at a point 50 m above the ground.

a) we need distance across arch at ground level, when $y = 0$.

$y = -0.025x^2 + 4.8x - 38.4$

$0 = -0.025x^2 + 4.8x - 38.4$

$a = -0.025 \quad b = 4.8 \quad c = -38.4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(4.8) \pm \sqrt{(4.8)^2 - 4(-0.025)(-38.4)}}{2(-0.025)}$$

b) We need distance across arch 50 m above ground, when $y = 50$.

$y = -0.025x^2 + 4.8x - 38.4$

$50 = -0.025x^2 + 4.8x - 38.4$

$-50 \qquad -50$

$0 = -0.025x^2 + 4.8x - 88.4$

$a = -0.025 \quad b = 4.8 \quad c = -88.4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

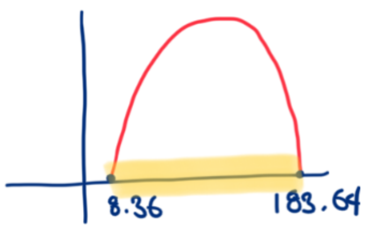
$$x = \frac{-(4.8) \pm \sqrt{(4.8)^2 - 4(-0.025)(-88.4)}}{2(-0.025)}$$

$$x = \frac{-4.8 \pm \sqrt{23.04 - 3.84}}{-0.05}$$

$$x = \frac{-4.8 \pm \sqrt{19.2}}{-0.05}$$

$x \doteq 8.36$ or $x \doteq 183.64$

So...



$183.64 - 8.36$
$= 175.28$

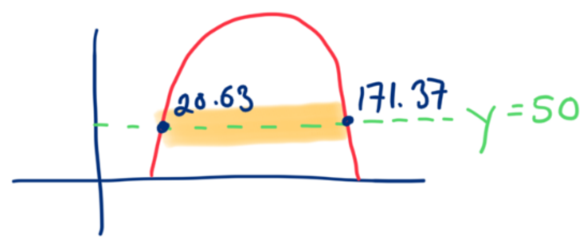
$\therefore$, at ground level, the horizontal distance from one side of the arch to the other is about 175.3 metres.

$$x = \frac{-4.8 \pm \sqrt{23.04 - 8.84}}{-0.05}$$

$$x = \frac{-4.8 \pm \sqrt{14.2}}{-0.05}$$

$x \doteq 20.63$ $x \doteq 171.37$

So...



$171.37 - 20.63$
$= 150.74$

$\therefore$, 50 m above the ground, the horizontal distance from one side of the arch to the other is about 150.7 m.

Example 2

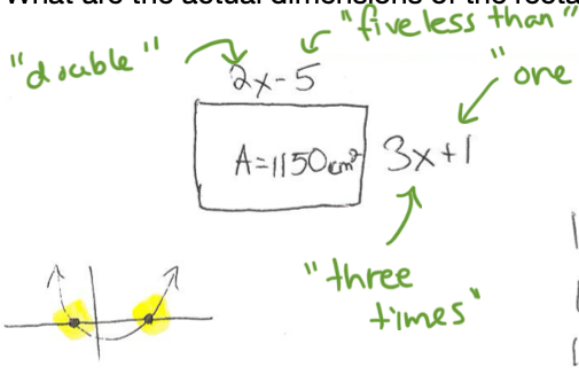
A rectangle has an area of 1150 cm^2.

The width is one more than three times an unknown measurement.

The length is five less than double an unknown measurement.

What are the actual dimensions of the rectangle?

Let x be the unknown measurement, in cm.



$A = lw$

$1150 = (2x-5)(3x+1)$ ← Sub in what we know.

$1150 = 6x^2 + 2x - 15x - 5$

$1150 = 6x^2 - 13x - 5$

$0 = 6x^2 - 13x - 1155$

To use the Quadratic formula... the expression must be in this form:

$0 = ax^2 + bx + c$

↑ $y=0$

Quadratic Formula (find the x-ints of a quadratic)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 6$

$b = -13$

$c = -1155$

Sub into the formula!

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(6)(-1155)}}{2(6)}$$

$$x = \frac{13 \pm \sqrt{169 + 27720}}{12}$$

$$x = \frac{13 \pm \sqrt{27889}}{12}$$

$$x = \frac{13 + \sqrt{27889}}{12} \quad \text{or} \quad x = \frac{13 - \sqrt{27889}}{12}$$

} exact answers (not rounded).

$x = 15$

$x = -12.83$ ← discard (you cannot have a negative length) → $3(-12)+1 = -35$

To finally answer question...

using $x = 15$

width	length
$3x+1$	$2x-5$
$= 3(15)+1 \rightarrow = 46$	$= 2(15)-5 \rightarrow = 25$

∴ the dimensions are 46cm by 25cm.