Opportunity to Learn – Standard Form to Vertex Form

1. Rewrite each relation in the form $y = a(x-h)^2 + k$ by completing the square.

   As you go, check that your work is correct using Desmos, by graphing both the original relation and the relation in $y = a(x-h)^2 + k$ form.

   a. $y = x^2 + 6x - 1$

   $\left(\frac{6}{2}\right)^2$
   $= 3^2$
   $= 9$

   $y = x^2 + 6x + 9 - 9 - 1$
   $y = (x+3)^2 - 10$

   b. $y = x^2 + 10x + 20$

   $y = x^2 + 10x + 25 - 25 + 20$
   $y = (x+5)^2 - 5$

   c. $y = x^2 - 6x - 4$

   $y = x^2 - 6x + 9 - 9 - 4$
   $y = (x-3)^2 - 13$

   d. $y = x^2 - 8x - 2$

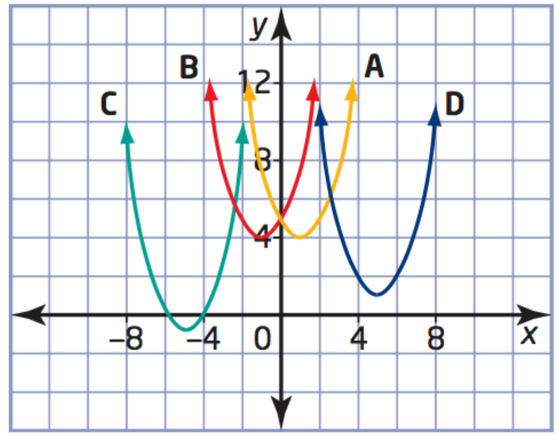
   $y = x^2 - 8x + 16 - 16 - 2$
   $y = (x-4)^2 - 18$

   e. $y = x^2 - 12x + 8$

   $y = x^2 - 12x + 36 - 36 + 8$
   $y = (x-6)^2 - 28$

2. Match each graph with the appropriate equation by filling the blank (A , B, C, or D).

   Provide point-form explanations for how you determined what equation matches each graph.



D $y = (x-5)^2 + 1$
largest shift to right

A $y = (x-1)^2 + 4$
process of elimination

B $y = (x+1)^2 + 4$
only equation with one unit shift left

C $y = (x+5)^2 - 1$
only equation with shift down

3. Find the vertex of each parabola.

   Sketch the graph, labelling the vertex, the axis of symmetry, and two other points on the curve.

a. $y = x^2 + 2x - 1$

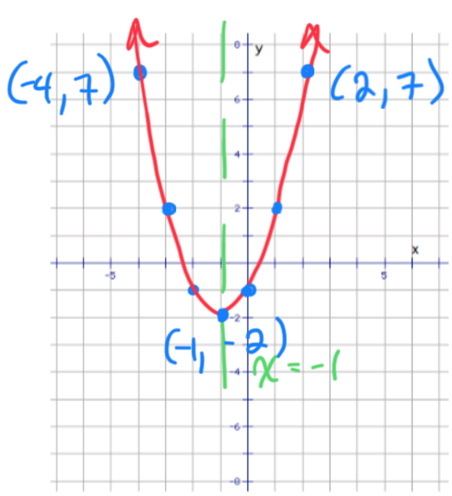
$y = \underline{x^2 + 2x} + 1 - 1 \quad - 1$

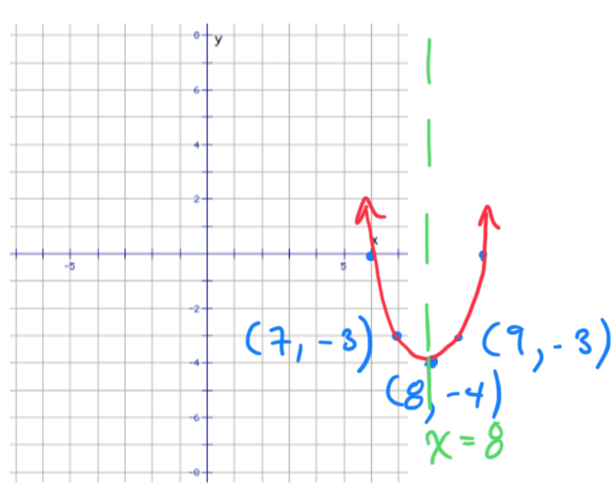
b. $y = x^2 - 16x + 60$

$y = \underline{x^2 - 16x + 64} - 64 \quad + 60$

$y = \underline{(x-8)^2} \quad - 64 + 60$

$y = (x-8)^2 - 4$





4. The path of a ball is modeled by the equation $y = -x^2 + 4x + 1$, where x is the horizontal distance travelled, in metres, and y is the height of the ball above the ground, in metres.

What is the maximum height of the ball, and at what horizontal distance does it occur?

y x

HINT: Consider the vertex…

$y = -x^2 + 4x + 1$ Factor out the -1

$y = -(x^2 - 4x - 1)$

$y = -(\underline{x^2 - 4x + 4} - 4 - 1)$

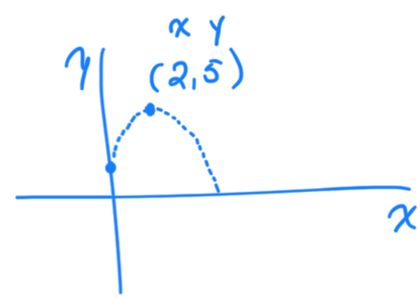
$y = -(\underline{(x-2)^2} - 4 - 1)$

$y = -((x-2)^2 - 5)$

$y = -(x-2)^2 + 5$ Distribute the -1 again!

vertex is $(2,5)$

parabola opens down

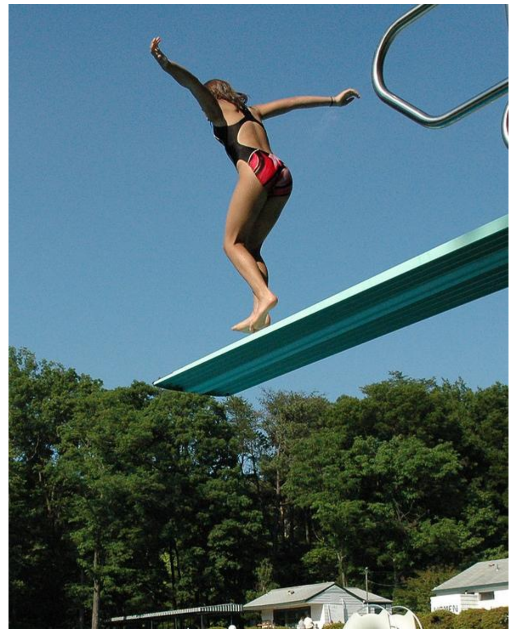


$\therefore$, the maximum height of the ball is 5 metres and this occurs after the ball has travelled 2 metres horizontally.

5. A diver dives from a 3-m board at a swimming pool.

   Her height, h, in metres above the surface of the pool, is related to her horizontal distance from the diving board, d, in metres, by the relation $h = -d^2 + 2d + 3$.

   What is the diver's maximum height?



$h = -d^2 + 2d + 3$

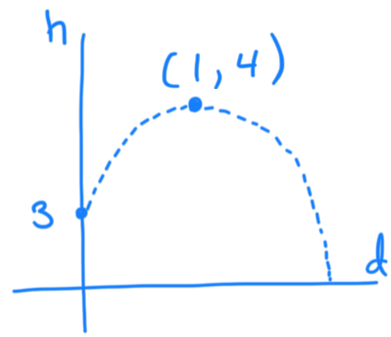
$h = -(d^2 - 2d - 3)$

$h = -(\underline{d^2 - 2d + 1} - 1 - 3)$

$h = -((d-1)^2 - 1 - 3)$

$h = -((d-1)^2 - 4)$

$h = -(d-1)^2 + 4$



∴, the diver's maximum height is 4 metres above the surface of the pool.