OTL – Standard Form to Vertex Form, Day 2

1. Rewrite each relation in the form $y = a(x-h)^2 + k$ by completing the square.

   As you go, check that your work is correct using Desmos, by graphing both the original relation and the relation in $y = a(x-h)^2 + k$ form.

   a. $y = -x^2 - 10x - 9$

a) $y = -x^2 - 10x - 9$ $\left(\frac{10}{2}\right)^2 = 25$

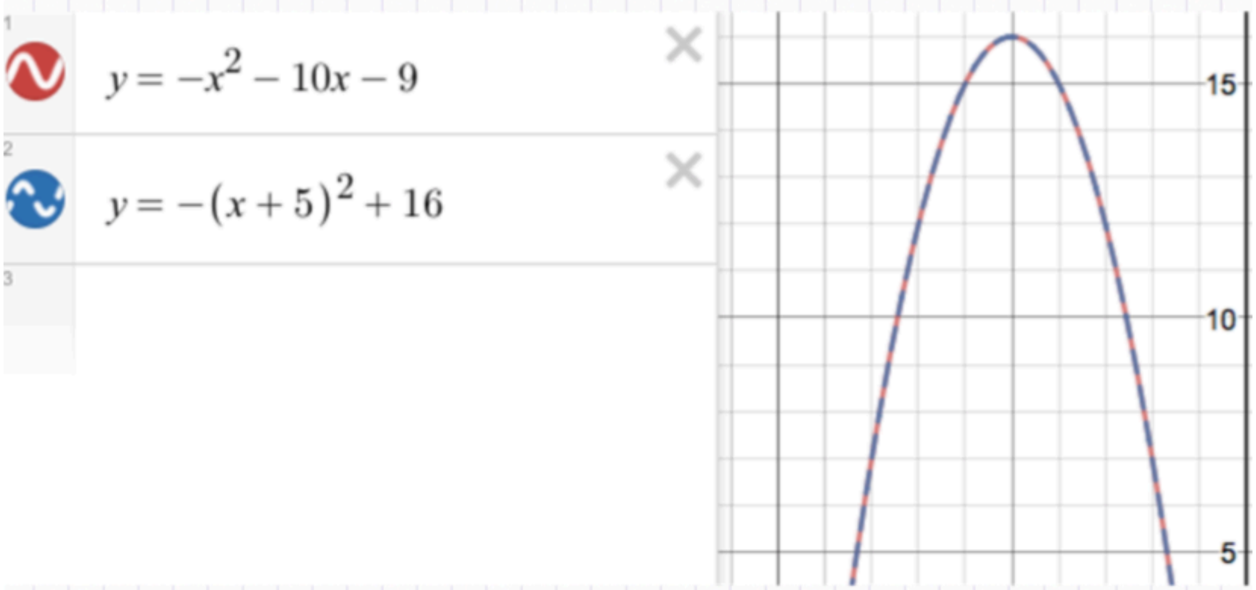
$y = -(x^2 + 10x + 9)$

$y = -(x^2 + 10x + 25 - 25 + 9)$

$y = -((x+5)^2 - 25 + 9)$

$y = -((x+5)^2 - 16)$

$y = -(x+5)^2 + 16$



b. $y = -x^2 + 14x - 50$

176 b) $y = -x^2 + 14x - 50$

$y = -(x^2 - 14x + 50)$ $\left(-\frac{14}{2}\right)^2 = 49$

$y = -(x^2 - 14x + 49 - 49 + 50)$

$y = -((x-7)^2 - 49 + 50)$

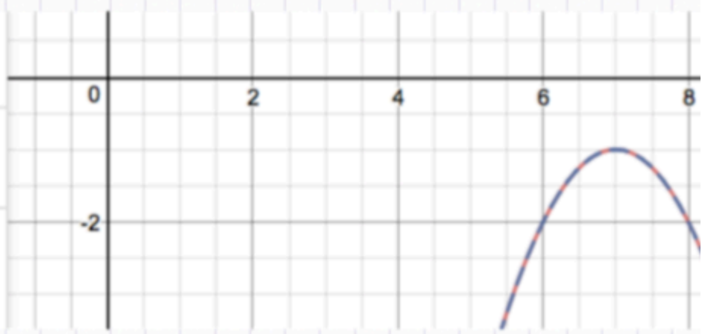
$y = -((x-7)^2 + 1)$

$y = -(x-7)^2 - 1$

1 $y = -x^2 + 14x - 50$

2 $y = -(x-7)^2 - 1$

3



c. $y = 2x^2 + 120x + 75$

176 c)

$$y = 2x^2 + 120x + 75$$

$$y = 2(x^2 + 60x) + 75 \qquad \left(\frac{60}{2}\right)^2 = 900$$

$$y = 2(x^2 + 60x + 900 - 900) + 75$$

$$y = 2((x + 30)^2 - 900) + 75)$$

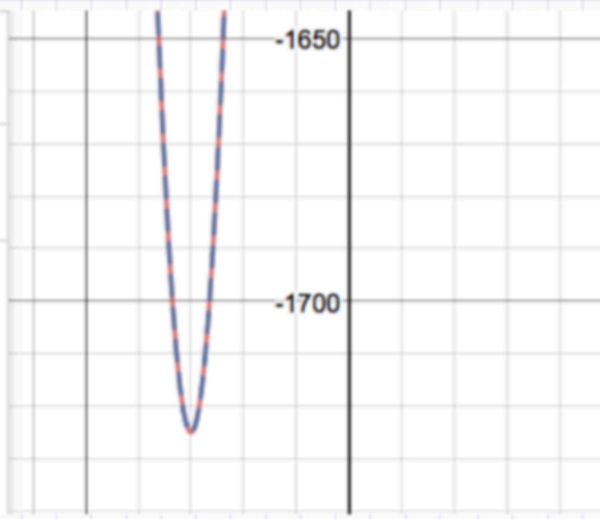
$$y = 2(x + 30)^2 - 1800 + 75$$

$$y = 2(x + 30)^2 - 1725$$

1 $y = 2x^2 + 120x + 75$

2 $y = 2(x + 30)^2 - 1725$

3



d. $y = 3x^2 - 24x + 10$

176 d)

$y = 3x^2 - 24x + 10$

$y = 3(x^2 - 8x) + 10$ $\quad \left(-\frac{8}{2}\right)^2 = 16$

$y = 3(x^2 - 8x + 16 - 16) + 10$

$y = 3((x-4)^2 - 16) + 10$

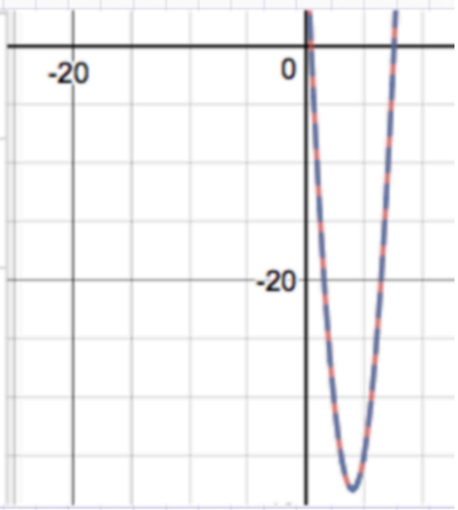
$y = 3(x-4)^2 - 48 + 10$

$y = 3(x-4)^2 - 38$

1 $y = 3x^2 - 24x + 10$

2 $y = 3(x-4)^2 - 38$

3



e. $y = -5x^2 - 200x - 120$

176e)

$y = -5x^2 - 200x - 120$

$y = -5(x^2 + 40x) - 120$ $\quad \left(\frac{40}{2}\right)^2 = 400$

$y = -5(x^2 + 40x + 400 - 400) - 120$

$y = -5((x+20)^2 - 400) - 120$

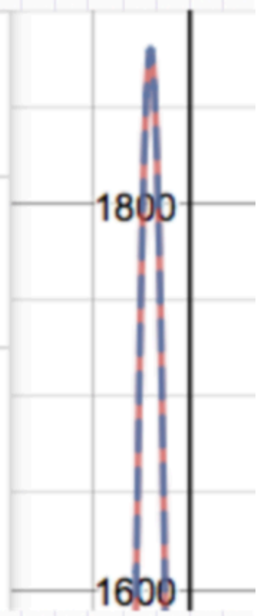
$y = -5(x+20)^2 + 2000 - 120$

$y = -5(x+20)^2 + 1880$

1 $y = -5x^2 - 200x - 120$

2 $y = -5(x+20)^2 + 1880$

3



2. Your friend says that:

   "Completing the square is a waste of time. You can always find the vertex of a quadratic by averaging the x-intercepts and then substituting back into the equation to get the y-value of the vertex."

   Is your friend speaking truth?

   That is, can you prove (or disprove) their conjecture?

   Create example(s) to support your position.

177.

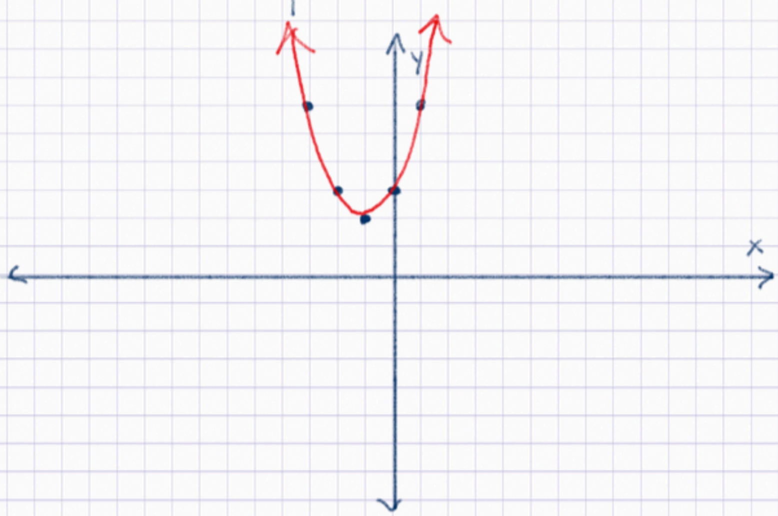
answers may vary

My friend's conjecture is not accurate.

Not all quadratic relations have x-intercepts, or roots.

For example:

$$y = x^2 + 2x + 3$$



Since the graph has no x-intercepts you cannot average the intercepts to obtain the vertex form equation:

$$y = (x+1)^2 + 2$$

3. Rewrite each relation in the form $y = a(x-h)^2 + k$ using whatever method you feel is most efficient.

   For each part, explain, in point-form, why you felt your approach was the most efficient.

   Check your work using Desmos.

   a. $y = x^2 + 17x + 30$

178

a) $y = x^2 + 17x + 30$

$y = (x+15)(x+2)$ ∴ x-ints are -15 and -2

• quick to factor by guess and check

∴ axis of symmetry is at: $\frac{-15-2}{2}$

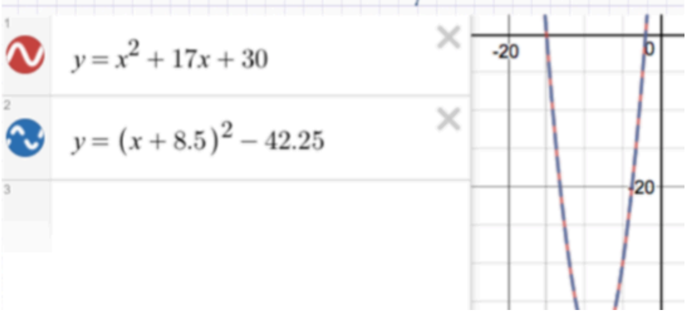
$= -8.5$

Sub to get y-value of vertex:

$y = (-8.5)^2 + 17(-8.5) + 30$

$y = -42.25$

∴ vertex form is: $y = (x+8.5)^2 - 42.25$



b. $y = 4x^2 - 16x + 12$

d)

$y = 4x^2 - 16x + 12$

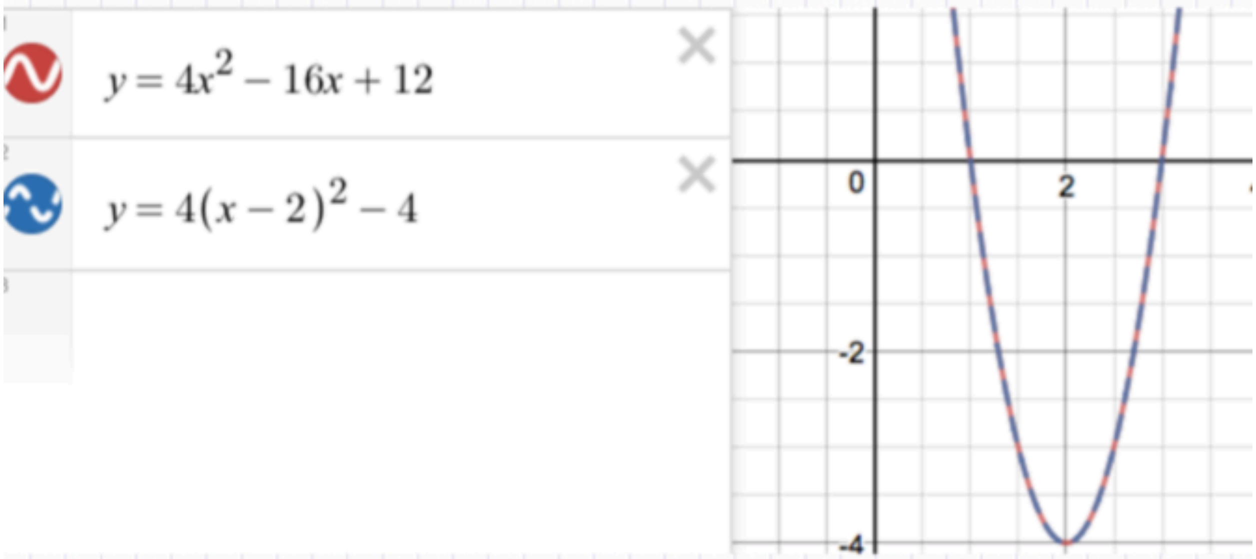
$y = 4(x^2 - 4x + 3)$ $\left(\frac{-4}{2}\right)^2 = 4$

$y = 4(\underline{x^2 - 4x + 4} - 4 + 3)$

$y = 4((\underline{x - 2)^2} - 4 + 3)$

$y = 4((x-2)^2 - 1)$

$y = 4(x-2)^2 - 4$



$$\left(\frac{\frac{3}{2}}{2}\right)^2 = \left(\frac{3}{2}\cdot\frac{1}{2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

c. $y = -2x^2 - 3x + 9$

$y = -2\left(x^2 + \frac{3}{2}x\right) + 9$ Factor out -2

$y = -2\left(x^2 + \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right) + 9$ Complete the square

$y = -2\left(\left(x + \frac{3}{4}\right)^2 - \frac{9}{16}\right) + 9$ Factor perfect square trinomial

$y = -2\left(x + \frac{3}{4}\right)^2 + \frac{18}{16} + 9$ Distribute.

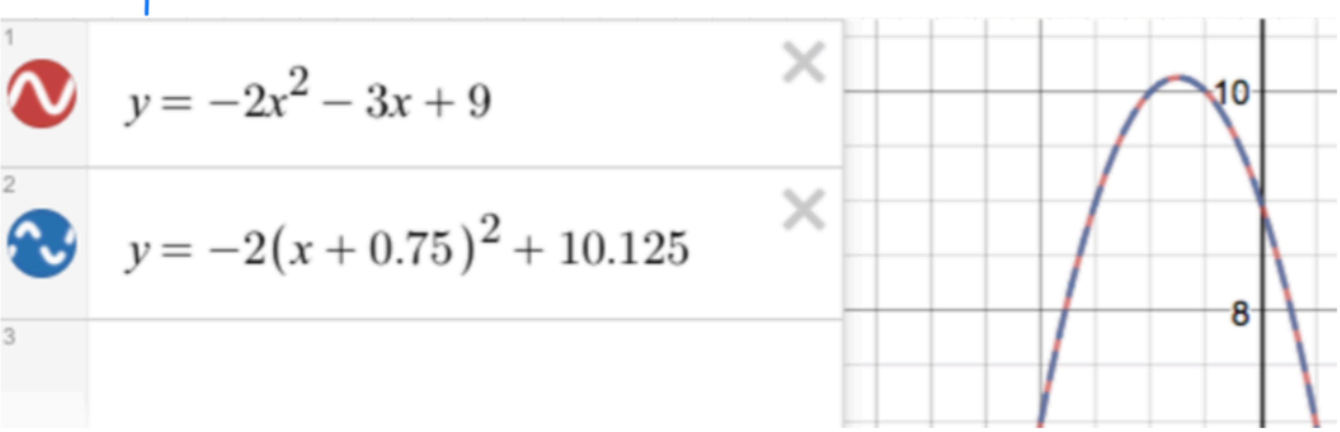
$y = -2\left(x + \frac{3}{4}\right)^2 + \frac{9}{8} + 9$ Simplify.

$y = -2\left(x + \frac{3}{4}\right)^2 + \frac{9}{8} + \frac{72}{8}$

$y = -2\left(x + \frac{3}{4}\right)^2 + \frac{81}{8}$

or

$y = -2(x + 0.75)^2 + 10.125$



d. $y = -5x^2 - 30x - 48$

e)

$y = -5x^2 - 30x - 48$ $\left(\frac{6}{2}\right)^2 = 9$

$y = -5(x^2 + 6x) - 48$

$y = -5(x^2 + 6x + 9 - 9) - 48$

$y = -5((x+3)^2 - 9) - 48$

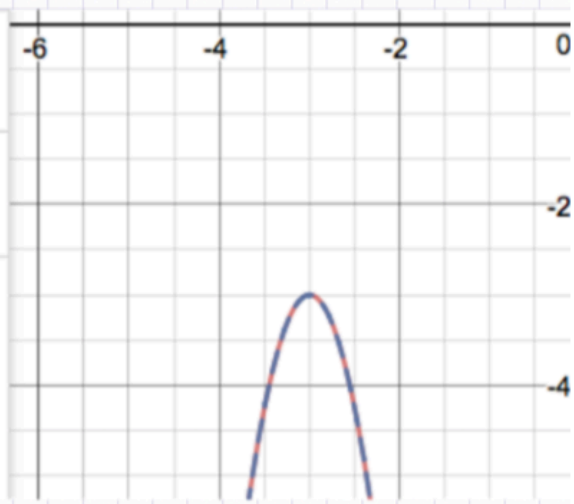
$y = -5(x+3)^2 + 45 - 48$

$y = -5(x+3)^2 - 3$

1 $y = -5x^2 - 30x - 48$

2 $y = -5(x+3)^2 - 3$

3



e. $y = -2x^2 - 3x + 7$

178 c)

$$y = -2x^2 - 3x + 7 \quad (-14)$$

1, -14
2, -7
-1, 14
-2, 7

(cannot be factored over the integers!)

· must complete the square.

$$y = -2\left(x^2 + \tfrac{3}{2}x\right) + 7 \qquad \left(\frac{\frac{3}{2}}{2}\right)^2$$

$$y = -2\left(x^2 + \tfrac{3}{2}x + \tfrac{9}{16} - \tfrac{9}{16}\right) + 7 \qquad = \left(\tfrac{3}{2}\left(\tfrac{1}{2}\right)\right)^2$$

$$y = -2\left(\left(x + \tfrac{3}{4}\right)^2 - \tfrac{9}{16}\right) + 7 \qquad = \left(\tfrac{3}{4}\right)^2 = \tfrac{9}{16}$$

$$y = -2\left(x + \tfrac{3}{4}\right)^2 + \tfrac{18}{16} + 7$$

$$y = -2\left(x + \tfrac{3}{4}\right)^2 + \tfrac{9}{8} + 7$$

$$y = -2\left(x + \tfrac{3}{4}\right)^2 + \tfrac{9}{8} + \tfrac{56}{8}$$

$$y = -2\left(x + \tfrac{3}{4}\right)^2 + \tfrac{65}{8}$$

1 $y = -2x^2 - 3x + 7$

2 $y = -2\left(x + \frac{3}{4}\right)^2 + \frac{65}{8}$

3

4. The cost, in dollars, of operating a machine per day is given by the formula $C = 2t^2 - 84t + 1025$, where t is the time, in hours, the machine operates, and C is the total cost, in dollars. What is the minimum cost of running the machine? For how many hours must the machine run to reach this minimum cost?

179.

need minimum cost
∴ we need the vertex!

$C = 2t^2 - 84t + 1025$

$\left(\frac{42}{2}\right)^2 = 441$

$C = 2(t^2 - 42t) + 1025$

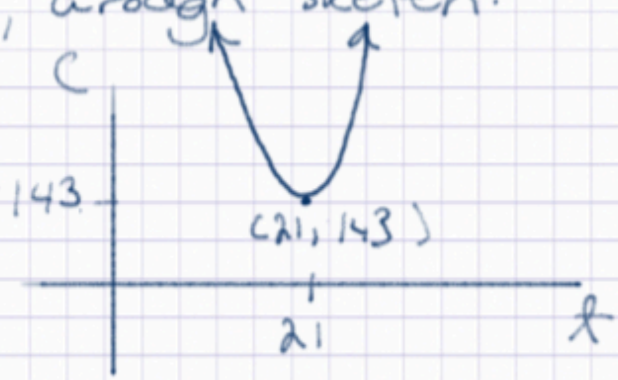
$C = 2(\underline{t^2 - 42t} \underline{+441} - 441) + 1025$

$C = 2((t-21)^2 - 441) + 1025$

$C = 2(t-21)^2 - 882 + 1025$

$C = 2(t-21)^2 + 143$

∴ a rough sketch:



∴ after 21 hours the minimum cost to run the machine is \$143

5. A quadratic relation has roots 0 and 6 and a maximum at (3, 4).

   Determine the equation of the relation using any method.

180.

One x-int at (0, 6).

Another x-int at (0, 0).

Maximum (vertex) at (3, 4).

$\therefore \quad y = a(x-h)^2 + k$

$y = a(x-3)^2 + 4$

Sub (0, 0) to get "a":

$0 = a(0-3)^2 + 4$

$0 = a(-3)^2 + 4$

$0 = 9a + 4$

$-4 \qquad\qquad -4$

$\frac{-4}{9} = \frac{9a}{9}$

$\frac{-4}{9} = a$

$\therefore$, equation is $y = -\frac{4}{9}(x-3)^2 + 4$