

**OTL – Solving Quadratic Equations by Factoring**

1. Find the roots of each quadratic equation.

- $0 = (x + 5)(x + 2)$
- $0 = (x - 3)(x + 4)$
- $0 = (x - 1)(x - 7)$
- $0 = x(x + 9)$
- $0 = (2x + 3)(3x - 5)$

a)  $0 = (x + 5)(x + 2)$

$$\begin{array}{l} \swarrow \qquad \searrow \\ x + 5 = 0 \qquad x + 2 = 0 \\ x + 5 - 5 = -5 \qquad x + 2 - 2 = -2 \\ x = -5 \qquad x = -2 \end{array}$$

$\therefore$  roots are  $x = -5$  or  $x = -2$

b)  $0 = (x - 3)(x + 4)$

$$\begin{array}{l} \swarrow \qquad \searrow \\ x - 3 = 0 \qquad x + 4 = 0 \\ x - 3 + 3 = +3 \qquad x + 4 - 4 = -4 \\ x = 3 \qquad x = -4 \end{array}$$

$\therefore$  roots are  $x = 3$  or  $x = -4$

c)  $0 = (x - 1)(x - 7)$

$$\begin{array}{l} \swarrow \qquad \searrow \\ x - 1 = 0 \qquad x - 7 = 0 \\ x - 1 + 1 = 1 \qquad x - 7 + 7 = 7 \\ x = 1 \qquad x = 7 \end{array}$$

$\therefore$  roots are  $x = 1$  or  $x = 7$

d)  $0 = x(x + 9)$

$$\begin{array}{l} \swarrow \qquad \searrow \\ x = 0 \qquad x + 9 = 0 \\ x + 9 - 9 = -9 \\ x = -9 \end{array}$$

$\therefore$  roots are  $x = 0$  or  $x = -9$

e)  $0 = (2x + 3)(3x - 5)$

$$\begin{array}{l} \swarrow \qquad \searrow \\ 2x + 3 = 0 \qquad 3x - 5 = 0 \\ 2x + 3 - 3 = -3 \qquad 3x - 5 + 5 = 5 \\ 2x = -3 \qquad 3x = 5 \\ \frac{2x}{2} = \frac{-3}{2} \qquad \frac{3x}{3} = \frac{5}{3} \\ x = -\frac{3}{2} \qquad x = \frac{5}{3} \end{array}$$

$\therefore$  roots are  $x = -\frac{3}{2}$  or  $x = \frac{5}{3}$

2. Find the roots of each quadratic equation.

- a.  $0 = x^2 + 8x + 12$
- b.  $0 = x^2 + 3x$
- c.  $x^2 + 5x = -4$
- d.  $x^2 = 7x$
- e.  $3x^2 + 24x + 45 = 0$

a)  $0 = x^2 + 8x + 12$   
 $0 = (x+6)(x+2)$

$\therefore$  roots are  $x = -6$   
 $\text{or } x = -2$

b)  $0 = x^2 + 3x$   
 $0 = x(x+3)$   
 $\downarrow \quad \uparrow$   
 $x=0 \quad x+3=0$   
 $x+3-3=-3$   
 $x=-3$

$\therefore$  roots are  $x = 0$   
 $\text{or } x = -3$

c)  $x^2 + 5x = -4$   
 $x^2 + 5x + 4 = -4 + 4$   
 $x^2 + 5x + 4 = 0$   
 $(x+4)(x+1) = 0$

$\therefore$  roots are  
 $x = -4$  or  
 $x = -1$ .

d)  $x^2 = 7x$   
 $x^2 - 7x = 7x - 7x$   
 $x^2 - 7x = 0$   
 $x(x-7) = 0$   
 $\downarrow \quad \downarrow$   
 $x=0 \quad x-7=0$   
 $x-7+7=7$   
 $x=7$

$\therefore$  roots are  $x = 0$   
 $\text{or } x = 7$

e)  $3x^2 + 24x + 45 = 0$

$$\frac{3x^2 + 24x + 45}{3} = \frac{0}{3}$$

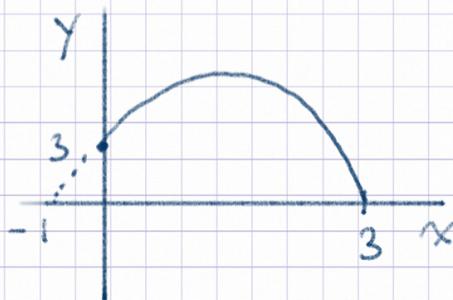
$$x^2 + 8x + 15 = 0$$
  
 $(x+5)(x+3) = 0$

$\therefore$  roots are  
 $x = -5$  or  
 $x = -3$

3. A basketball is tossed from the top of a 3-m wall.

The path of the basketball is defined by the relation  $y = -x^2 + 2x + 3$ , where  $x$  represents the horizontal distance travelled, in metres, and  $y$  represents the height, in metres, above the ground.

How far has the basketball travelled horizontally when it lands on the ground?



"lands on the ground"  
implies we need  
to find the roots...  
ie.  $x$ -values at  
 $y = 0$

$$0 = -x^2 + 2x + 3$$

$$\frac{0}{-1} = \frac{-x^2 + 2x + 3}{-1}$$

$$0 = x^2 - 2x - 3$$

$$0 = (x + 1)(x - 3)$$

$\therefore$  roots are  $x = -1$

$$\text{or } x = 3$$

$\therefore$  the basketball  
has travelled  
3 m horizontally

4. A rectangle has dimensions  $x + 10$  and  $2x - 3$ .

Determine the value of  $x$  that gives an area of  $54 \text{ cm}^2$ .

Remember that the area of a rectangle is defined by  $A = lw$ .

$$x + 10$$



$$2x - 3$$

OK...  $A = lw$ .

Let's use the info given and substitute  $g$  into the area formula.

$$54 = (x + 10)(2x - 3)$$

$$54 = 2x^2 - 3x + 20x - 30$$

$$54 = 2x^2 + 17x - 30$$

$$54 - 54 = 2x^2 + 17x - 30 - 54$$

$$0 = 2x^2 + 17x - 84$$

Now, factor to find intercepts or roots.

$$-168$$

$$0 = 2x^2 - 7x + 24$$

$$-7, 24$$

$$0 = x(2x - 7) + 12(2x - 7)$$

$$0 = (2x - 7)(x + 12)$$

↙

$$\begin{aligned} 2x - 7 &= 0 \\ 2x - 7 + 7 &= 7 \\ 2x &= 7 \\ \frac{2x}{2} &= \frac{7}{2} \\ x &= \frac{7}{2} \\ x &= 3.5 \end{aligned}$$

↗

$$\begin{aligned} x + 12 &= 0 \\ x + 12 - 12 &= -12 \\ x &= -12 \end{aligned}$$

∴ roots are  $x = 3.5$   
or  $x = -12$

Dimensions of a rectangle cannot be negative though.

discard this root

∴  $x = 3.5$  ∴ length is 13.5 m.  
and width is 4 m

Check:  $(13.5)(4) = 54$

5. Write a quadratic equation, in standard form, that has roots of 5 and -8.

answers may vary

$$y = a(x - r)(x - s)$$

Let  $a = 1$ .

$$y = 1(x - 5)(x - (-8))$$

$$y = (x - 5)(x + 8)$$

factored or intercepts form

$$y = x^2 + 9x - 5x - 40$$

$$y = x^2 + 3x - 40$$

expand to get to standard form.

6. Find the  $x$ -intercepts for each quadratic relation.

- $y = x^2 + 5x + 6$
- $y = x^2 + 9x$
- $y = x^2 + 9x - 36$
- $y = 4x^2 + 20x + 9$
- $y = 3x^2 - 13x + 4$

a)  $y = x^2 + 5x + 6$

$$0 = (x+2)(x+3)$$

$\therefore x$ -intercepts  
are  $x = -2$  or  
 $x = -3$

b)  $y = x^2 + 9x$

$$0 = x(x+9)$$

$$\begin{array}{c} \swarrow \\ x=0 \end{array}$$

$$\begin{array}{c} \searrow \\ x+9=0 \\ x+9-9=-9 \\ x=-9 \end{array}$$

$\therefore x$ -intercepts are  
 $x = 0$  or  
 $x = -9$ .

c)  $y = x^2 + 9x - 36$

$$0 = (x+12)(x-3)$$

$\therefore x$ -intercepts  
are  $x = -12$   
or  $x = 3$

d)  $y = \boxed{4x^2 + 20x + 9}$  (36)

$$0 = 4x^2 + 18x + 2x + 9 \quad 18, 2$$

$$0 = 2x(2x+9) + 1(2x+9)$$

$$0 = (2x+9)(2x+1)$$

$$\begin{array}{c} \swarrow \\ 0 = 2x+9 \end{array}$$

$$-9 = 2x+9-9$$

$$-9 = 2x$$

$$-\frac{9}{2} = \frac{2x}{2}$$

$$-\frac{9}{2} = x$$

$$\begin{array}{c} \searrow \\ 2x+1=0 \end{array}$$

$$2x+1-1=-1$$

$$2x = -1$$

$$\frac{2x}{2} = -\frac{1}{2}$$

$$x = -\frac{1}{2}$$

$\therefore x$ -intercepts are  $x = -\frac{1}{2}$  or  
 $x = -\frac{9}{2}$

e)  $y = \boxed{3x^2 - 12x + 4}$  (12)

$$-12, 7$$

$$0 = 3x^2 - 12x - x + 4$$

$$0 = 3x(x-4) - 1(x-4)$$

$$0 = (x-4)(3x-1)$$

$\therefore x$ -intercepts are  $x = 4$

$$\text{or } x = \frac{1}{3}$$