

OTL – Solving Quadratic Equations by Factoring

1. Find the roots of each quadratic equation.

a. $0 = (x + 5)(x + 2)$

b. $0 = (x - 3)(x + 4)$

c. $0 = (x - 1)(x - 7)$

d. $0 = x(x + 9)$

e. $0 = (2x + 3)(3x - 5)$

a) $0 = (x + 5)(x + 2)$

\swarrow $x + 5 = 0$ \searrow $x + 2 = 0$
 $x + 5 - 5 = -5$ $x + 2 - 2 = -2$
 $x = -5$ $x = -2$

\therefore roots are $x = -5$ or $x = -2$

b) $0 = (x - 3)(x + 4)$

\swarrow $x - 3 = 0$ \searrow $x + 4 = 0$
 $x - 3 + 3 = +3$ $x + 4 - 4 = -4$
 $x = 3$ $x = -4$

\therefore roots are $x = 3$ or $x = -4$

c) $0 = (x - 1)(x - 7)$

\swarrow $x - 1 = 0$ \searrow $x - 7 = 0$
 $x - 1 + 1 = 1$ $x - 7 + 7 = 7$
 $x = 1$ $x = 7$

\therefore roots are $x = 1$ or $x = 7$

d) $0 = x(x + 9)$

\swarrow $x = 0$ \searrow $x + 9 = 0$
 $x + 9 - 9 = -9$
 $x = -9$

\therefore roots are $x = 0$ or $x = -9$

e) $0 = (2x + 3)(3x - 5)$

\swarrow $2x + 3 = 0$ \searrow $3x - 5 = 0$
 $2x + 3 - 3 = -3$ $3x - 5 + 5 = 5$
 $2x = -3$ $3x = 5$
 $\frac{2x}{2} = \frac{-3}{2}$ $\frac{3x}{3} = \frac{5}{3}$
 $x = -\frac{3}{2}$ $x = \frac{5}{3}$

\therefore roots are $x = -\frac{3}{2}$ or $x = \frac{5}{3}$

2. Find the roots of each quadratic equation.

a. $0 = x^2 + 8x + 12$

b. $0 = x^2 + 3x$

c. $x^2 + 5x = -4$

d. $x^2 = 7x$

e. $3x^2 + 24x + 45 = 0$

a) $0 = x^2 + 8x + 12$
 $0 = (x+6)(x+2)$

\therefore roots are $x = -6$
 or $x = -2$

b) $0 = x^2 + 3x$
 $0 = x(x+3)$

\downarrow
 $x = 0$

\downarrow
 $x+3=0$
 $x+3-3=-3$
 $x=-3$

\therefore roots are $x = 0$
 or $x = -3$

c) $x^2 + 5x = -4$

$x^2 + 5x + 4 = -4 + 4$

$x^2 + 5x + 4 = 0$

$(x+4)(x+1) = 0$

\therefore roots are
 $x = -4$ or
 $x = -1$

d) $x^2 = 7x$

$x^2 - 7x = 7x - 7x$

$x^2 - 7x = 0$

$x(x-7) = 0$

\downarrow
 $x = 0$

\downarrow
 $x-7=0$
 $x-7+7=7$
 $x=7$

\therefore roots are $x = 0$
 or $x = 7$

e) $3x^2 + 24x + 45 = 0$

$\frac{3x^2 + 24x + 45}{3} = \frac{0}{3}$

$x^2 + 8x + 15 = 0$

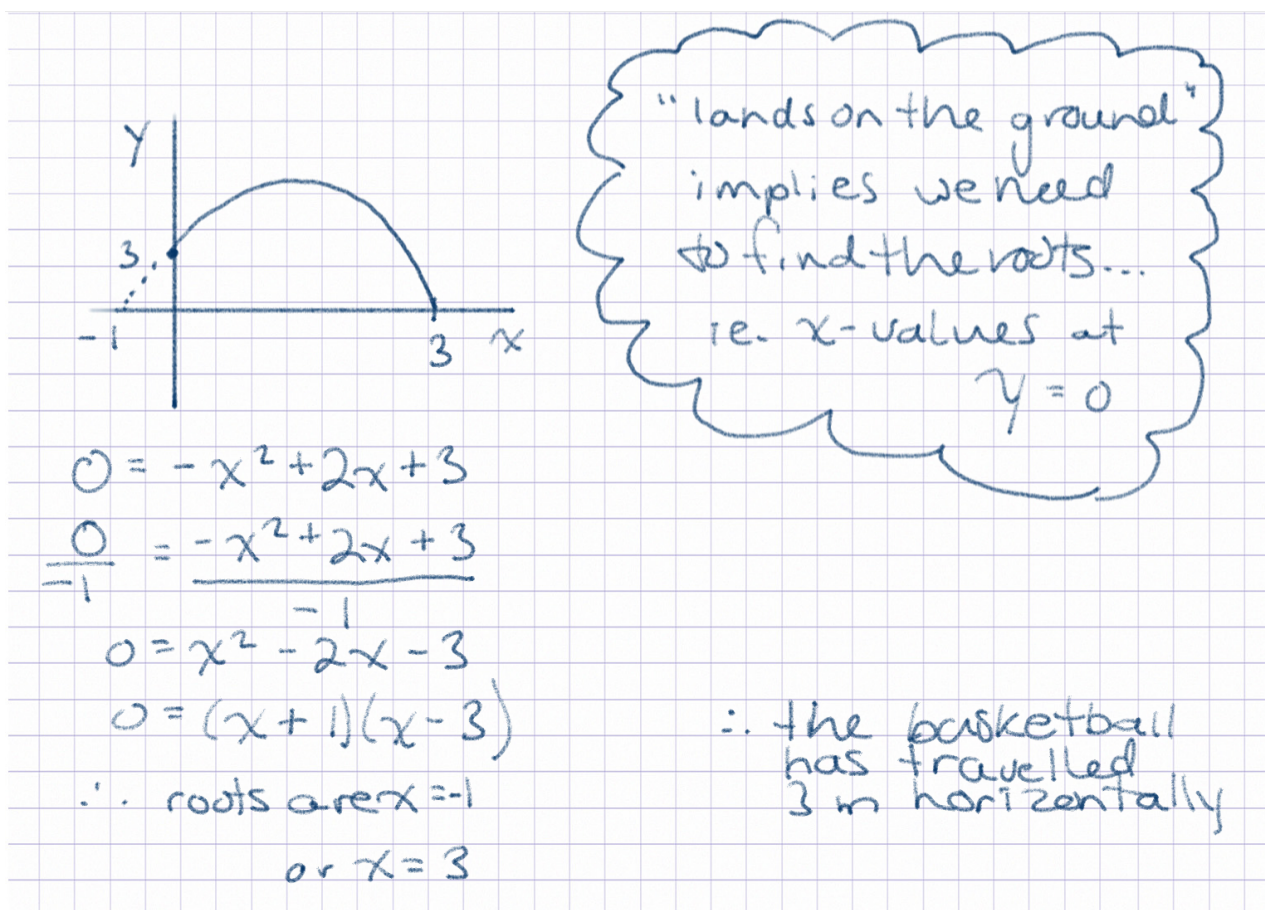
$(x+5)(x+3) = 0$

\therefore roots are
 $x = -5$ or
 $x = -3$

3. A basketball is tossed from the top of a 3-m wall.

The path of the basketball is defined by the relation $y = -x^2 + 2x + 3$, where x represents the horizontal distance travelled, in metres, and y represents the height, in metres, above the ground.

How far has the basketball travelled horizontally when it lands on the ground?



4. A rectangle has dimensions $x + 10$ and $2x - 3$.

Determine the value of x that gives an area of 54 cm^2 .

Remember that the area of a rectangle is defined by $A = lw$.

$x + 10$



$2x - 3$

OK... $A = lw$. Let's use the info given and substitute into the area formula.

$$54 = (x + 10)(2x - 3)$$

$$54 = 2x^2 - 3x + 20x - 30$$

$$54 = 2x^2 + 17x - 30$$

$$54 - 54 = 2x^2 + 17x - 30 - 54$$

$$0 = 2x^2 + 17x - 84$$

Now factor to find intercepts or roots.

$$0 = 2x^2 - 7x + 24x - 84$$

$$0 = x(2x - 7) + 12(2x - 7)$$

$$0 = (2x - 7)(x + 12)$$

$2x - 7 = 0$
 $2x - 7 + 7 = 7$
 $2x = 7$
 $\frac{2x}{2} = \frac{7}{2}$
 $x = \frac{7}{2}$
 $x = 3.5$

$x + 12 = 0$
 $x + 12 - 12 = -12$
 $x = -12$

∴ roots are $x = 3.5$ or $x = -12$

Dimensions of a rectangle cannot be negative though.

∴ $x = 3.5$ ∴ length is 13.5 m. and width is 4 m

Check: $(13.5)(4) = 54$

discarded this root

5. Write a quadratic equation, in standard form, that has roots of 5 and -8.

answers may vary

$$y = a(x - r)(x - s)$$

Let $a = 1$.

$$y = 1(x - (5))(x - (-8))$$

$$y = (x - 5)(x + 8)$$

factored or intercepts form

$$y = x^2 + 8x - 5x - 40$$

Expand to get to standard form.

$$y = x^2 + 3x - 40$$

6. Find the x -intercepts for each quadratic relation.

a. $y = x^2 + 5x + 6$

b. $y = x^2 + 9x$

c. $y = x^2 + 9x - 36$

d. $y = 4x^2 + 20x + 9$

e. $y = 3x^2 - 13x + 4$

a) $y = x^2 + 5x + 6$
 $0 = (x+2)(x+3)$
 $\therefore x$ -intercepts are $x = -2$ or $x = -3$

b) $y = x^2 + 9x$
 $0 = x(x+9)$
 $\downarrow \quad \searrow$
 $x=0 \quad x+9=0$
 $\quad \quad x+9-9=-9$
 $\quad \quad x=-9$
 $\therefore x$ -intercepts are $x=0$ or $x=-9$.

c) $y = x^2 + 9x - 36$
 $0 = (x+12)(x-3)$
 $\therefore x$ -intercepts are $x = -12$ or $x = 3$

d) $y = 4x^2 + 20x + 9$ (36)
 $0 = 4x^2 + 18x + 2x + 9$ 18, 2
 $0 = 2x(2x+9) + 1(2x+9)$
 $0 = (2x+9)(2x+1)$
 $\downarrow \quad \searrow$
 $0 = 2x+9 \quad 2x+1=0$
 $-9 = 2x+9-9 \quad 2x+1-1=-1$
 $-9 = 2x \quad 2x = -1$
 $-\frac{9}{2} = \frac{2x}{2} \quad \frac{2x}{2} = -\frac{1}{2}$
 $-\frac{9}{2} = x \quad x = -\frac{1}{2}$
 $\therefore x$ -intercepts are $x = -\frac{9}{2}$ or $x = -\frac{1}{2}$

e) $y = 3x^2 - 13x + 4$ (12)
 $0 = 3x^2 - 12x - x + 4$ -12, 1
 $0 = 3x(x-4) - 1(x-4)$
 $0 = (x-4)(3x-1)$
 $\therefore x$ -intercepts are $x = 4$ or $x = \frac{1}{3}$