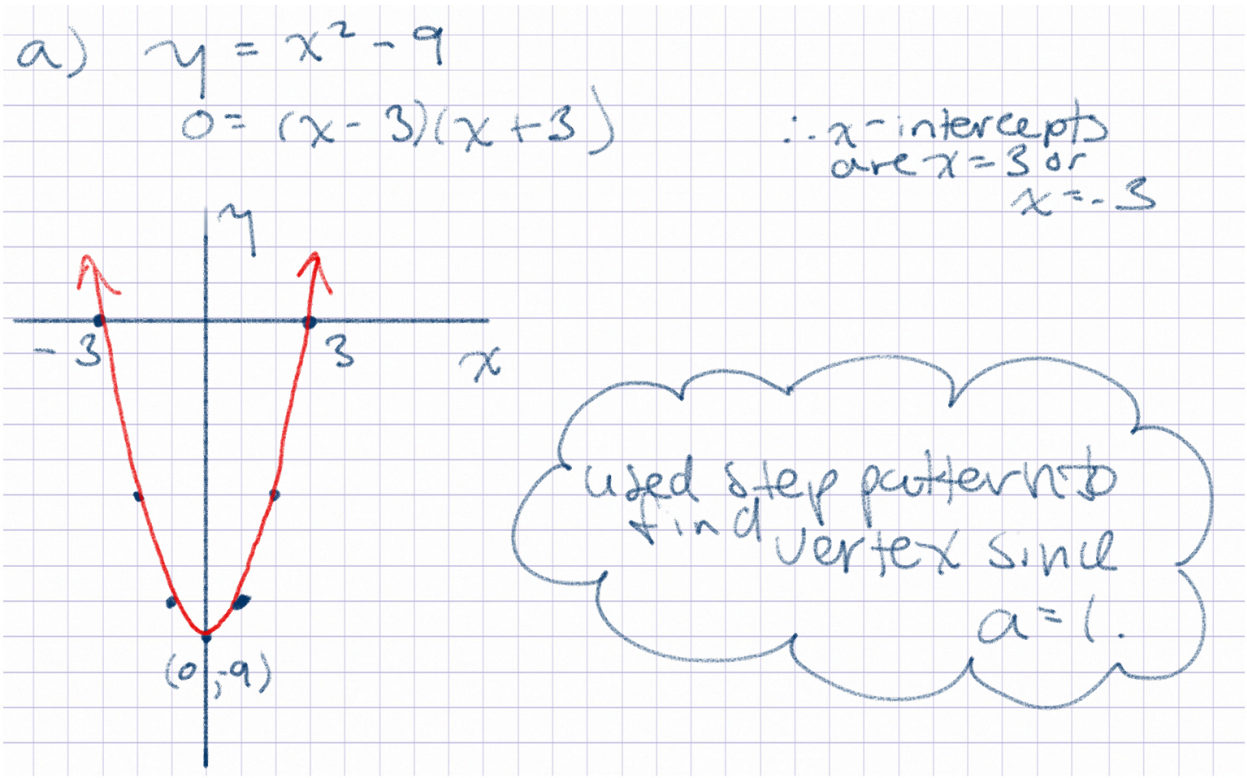
OTL – Graphing Quadratics Using the Intercepts and Vertex

1. Find the x-intercepts and the vertex of each parabola. Then, sketch it's graph on a Cartesian plane.

   a. $y = x^2 - 9$
   b. $y = -x^2 + 10x - 9$
   c. $y = x^2 - 12x + 36$



b) $y = -x^2 + 10x - 9$

$\frac{0}{-1} = \frac{-x^2 + 10x - 9}{-1}$

$0 = x^2 - 10x + 9$

$0 = (x-9)(x-1)$

$\therefore$ roots are $x = 9$ or $x = 1$

Now average zeroes to get x-value of vertex:

$\frac{9+1}{2}$

$= \frac{10}{2}$

$= 5$

Now substitute to get y-value of vertex:

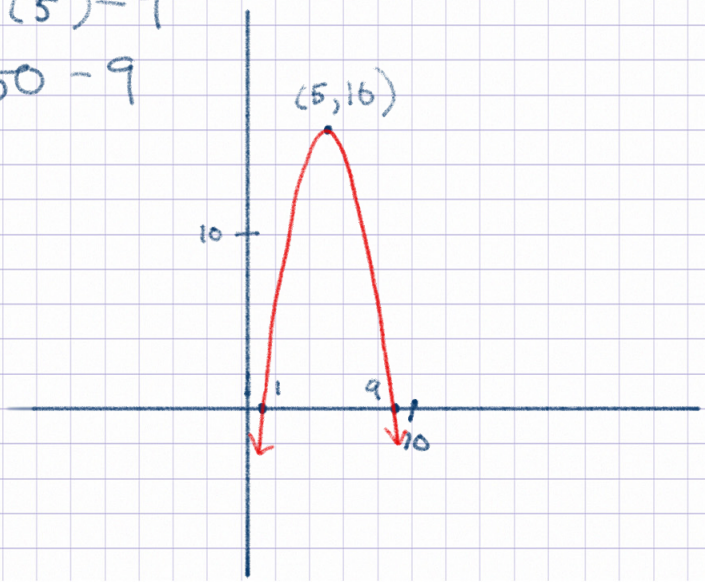
$y = -x^2 + 10x - 9$ Sub $x = 5$.

$y = -(5)^2 + 10(5) - 9$

$y = -(25) + 50 - 9$

$y = 25 - 9$

$y = 16$

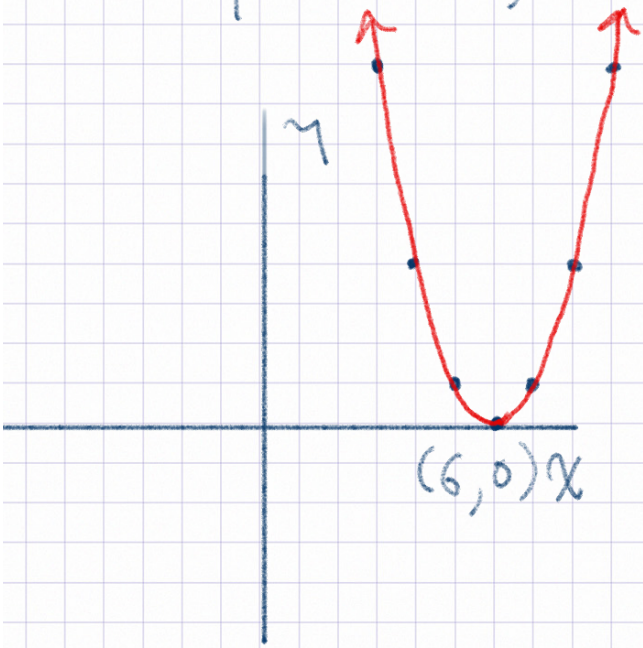


c) $y = x^2 - 12x + 36$

$y = (x-6)(x-0)$

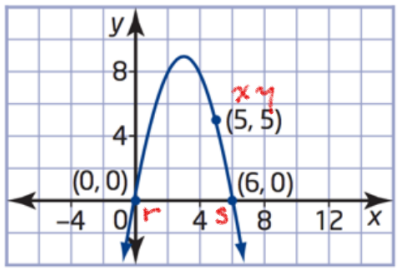
$y = (x-6)^2$

∴ x-intercepts are $x = 6$ or $x = 6$



2. Write an equation in $y = ax^2 + bx + c$ form (standard form) to represent each parabola.

a.



$y = a(x-r)(x-s)$

$5 = a(5-0)(5-6)$

$5 = a(5)(-1)$

$5 = -5a$

$\frac{5}{-5} = \frac{-5a}{-5}$

$-1 = a$

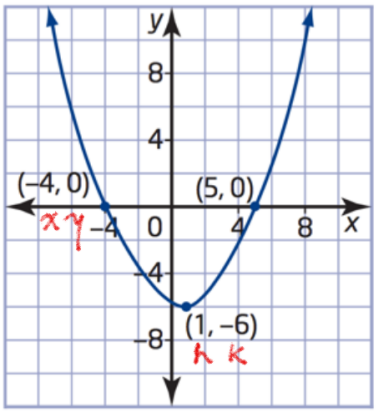
Substitute:

$y = -1(x-0)(x-6)$

$y = -x(x-6)$

$y = -x^2 + 6x$

b.



$y = a(x-h)^2 + k$

$0 = a(-4-(1))^2 + (-6)$

$0 = a(-4-1)^2 - 6$

$0 = a(-5)^2 - 6$

$0 = 25a - 6$

$0+6 = 25a - 6 + 6$

$\frac{6}{25} = \frac{25a}{25}$

$\frac{6}{25} = a$

Substitute:

$y = \frac{6}{25}(x-1)^2 - 6$

$y = \frac{6}{25}(x^2 - 2x + 1) - 6$

$y = \frac{6}{25}x^2 - \frac{12}{25}x + \frac{6}{25} - \frac{150}{25}$

$y = \frac{6}{25}x^2 - \frac{12}{25}x - \frac{144}{25}$

3. The path of a toy rocket is defined by the relation $y = -3x^2 + 11x + 4$, where x is the horizontal distance, in metres, travelled and y is the height, in metres, above the ground.

   a. Determine the zeros of the relation.

   b. For what values of x is the relation valid?

   c. How far has the rocket travelled horizontally when it lands on the ground?

   d. What is the maximum height of the rocket above the ground, to the nearest hundredth of a metre?

a) $0 = -3x^2 + 11x + 4$ (-12) 12, -1

$0 = -3x^2 + 12x - x + 4$

$0 = -3x(x-4) - 1(x-4)$

$0 = (x-4)(-3x-1)$

$x - 4 = 0$ $\quad$ $-3x - 1 = 0$

$x - 4 + 4 = 4$ $\quad$ $-3x - 1 + 1 = 1$

$x = 4$ $\quad$ $-3x = 1$

$\frac{-3x}{-3} = \frac{1}{-3}$

$x = -\frac{1}{3}$

b) x represents horizontal distance. We discard $x = -\frac{1}{3}$ because the rocket would not travel backwards after being launched.

c) 4 metres.

d) Average the zeros to get x-value of vertex:

$$\frac{-0.3333 + 4}{2}$$

$$\doteq 1.833$$

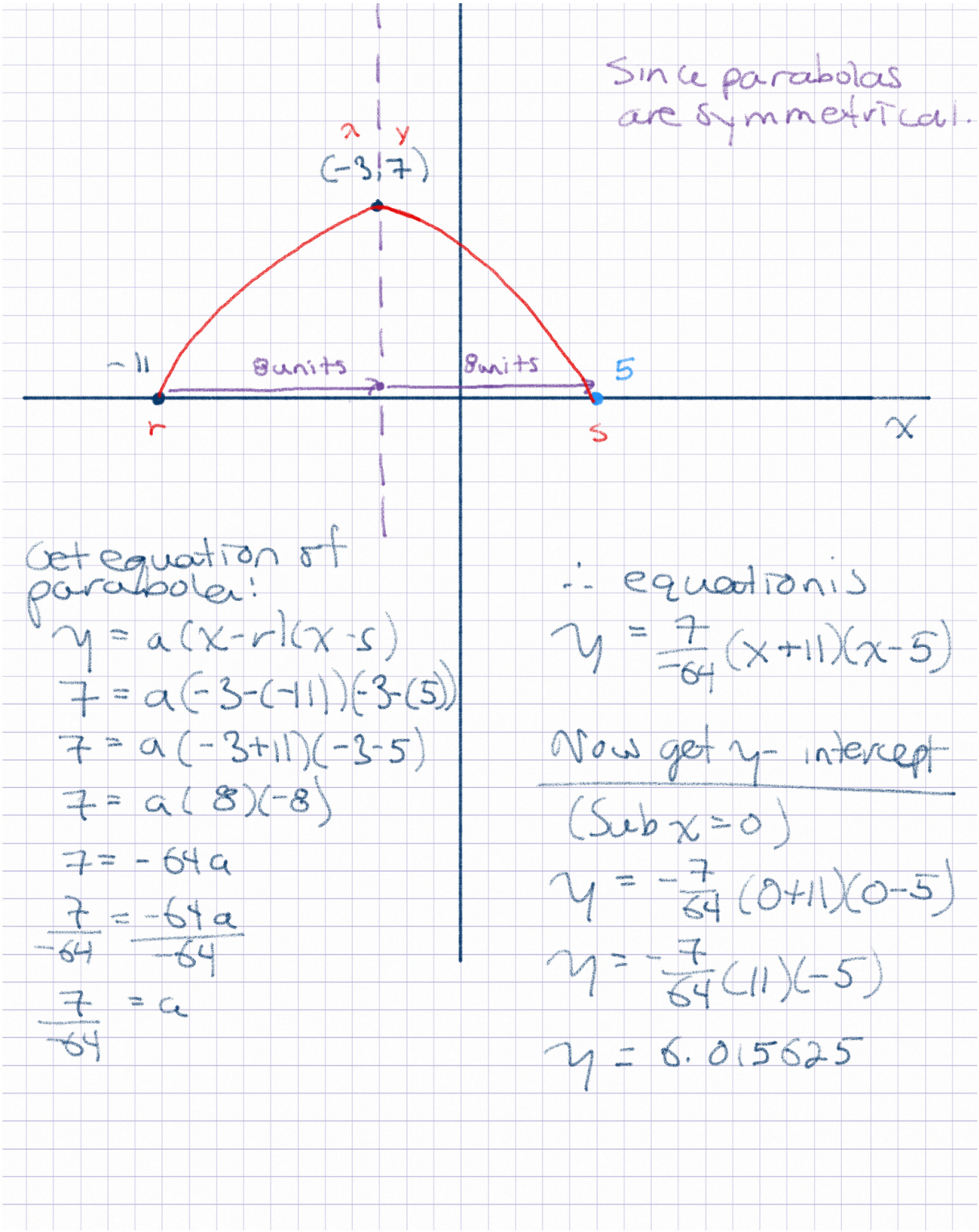
Sub to get y-value of vertex:

$$y = -3x^2 + 11x + 4$$
$$y = -3(1.833)^2 + 11(1.833) + 4$$
$$y \doteq 14.08$$

∴ the rocket went 14.08 metres in the air, approximately.

4. A parabola has a vertex at $(-3,\ 7)$, and one of it's x-intercepts is -11.

   Show how you can determine the other x-intercept and the y-intercept of the parabola.



5. What must be true for a parabola to have only one x intercept?

   Explain, using an example with words, equations, and a graph.

For a parabola to have only one x-intercept, it's vertex must be on the x-axis.

Example... $y = (x+2)^2$