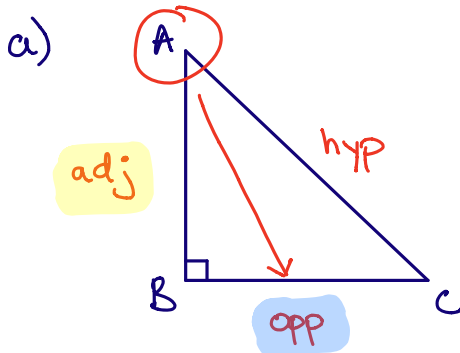


More Applications of the Primary Trigonometric Ratios

Example 1

$\triangle ABC$ has $\angle B = 90^\circ$.

- a) At what measure of $\angle A$ will $\sin A = \cos A$?
- b) What are the exact values of sine ratio and cosine ratio, in lowest terms, in this situation?



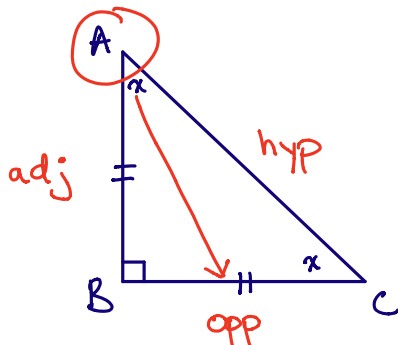
$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

Hmm... the hypotenuse will always be the same in both ratios (since it is literally the same side of the triangle).

In order for $\sin A = \cos A$, the side opposite $\angle A$ and the side adjacent to $\angle A$ would have to be the same length.

So:



If $\overline{AB} = \overline{BC}$ then $\angle A = \angle C$, by Isosceles Triangle Theorem.

$$\begin{aligned} \text{Then: } \angle A + \angle B + \angle C &= 180^\circ \\ x + 90 + x &= 180^\circ \\ 2x + 90 &= 180^\circ \\ -90 \quad -90 & \\ \hline 2x &= 90 \\ \frac{2x}{2} &= \frac{90}{2} \\ x &= 45^\circ \end{aligned}$$

\therefore , for $\sin A = \cos A$, $\angle A = 45^\circ$

b) Let $AB = 1$. Then $BC = 1$.

Now:

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\begin{aligned} \text{Then } AC^2 &= AB^2 + BC^2 \\ AC^2 &= (1)^2 + (1)^2 \\ AC^2 &= 1 + 1 \\ AC^2 &= 2 \end{aligned}$$

$$\pm \sqrt{AC^2} = \pm \sqrt{2}$$

$$AC = \sqrt{2} \quad (\text{since a length cannot be negative})$$

Opportunity to Learn

1. In $\triangle ABC$, $a = 12$ cm, $b = 10$ cm, and $\angle A = 45^\circ$.

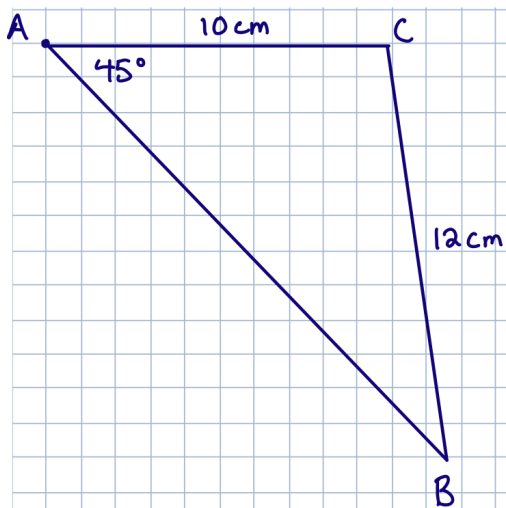
- a) Determine the exact length of side c .

HINTS:

Consider adding something to the diagram so that you can use primary trig ratios.

Think about using a proportion (see your answer to example 1 above).

- b) Determine the measure of $\angle C$, to the nearest whole degree.



2. If the shadow of a building increases by 10 meters when the angle of elevation of the sun rays decreases from 70° to 60° , what is the height of the building?

HINTS:

The start of this solution is given below; add something to the diagram given.

What equations can you set up? Can you combine equations to find unknown values?

