More Applications of the Primary Trigonometric Ratios

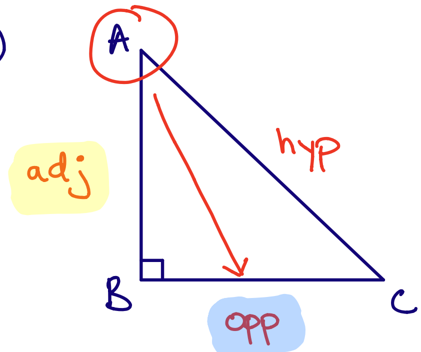
Example 1

$\triangle ABC$ has $\angle B = 90°$.

a) At what measure of $\angle A$ will sin $A =$ cos A?

b) What are the exact values of sine ratio and cosine ratio, in lowest terms, in this situation?

a)

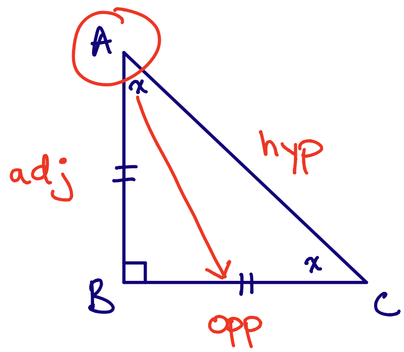


$\sin A = \frac{opp}{hyp}$ $\cos A = \frac{adj}{hyp}$

Hmm... the hypotenuse will always be the same in both ratios (since it is literally the same side of the triangle.

In order for $\sin A = \cos A$, the side opposite $\angle A$ and the side adjacent to $\angle A$ would have to be the same length.

So:



If $\overline{AB} = \overline{BC}$ then $\angle A = \angle C$, by Isosceles Triangle Theorem.

Then: $\angle A + \angle B + \angle C = 180°$

$x + 90 + x = 180°$

$2x + 90 = 180°$

$-90 \quad -90$

$2x = 90$

$\frac{2x}{2} = \frac{90}{2}$

$x = 45°$

$\therefore$, for $\sin A = \cos A$, $\angle A = 45°$

b) Let $AB = 1$. Then $BC = 1$.

Now:

$\sin A = \frac{opp}{hyp}$ $\cos A = \frac{adj}{hyp}$

$\sin 45° = \frac{1}{\sqrt{2}}$ $\cos 45° = \frac{1}{\sqrt{2}}$

Then $AC^2 = AB^2 + BC^2$

$AC^2 = (1)^2 + (1)^2$

$AC^2 = 1 + 1$

$AC^2 = 2$

$\pm\sqrt{AC^2} = \pm\sqrt{2}$

$AC = \sqrt{2}$ (since a length cannot be negative)

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Opportunity to Learn

1. In $\triangle ABC$, $a = 12$ cm, $b = 10$ cm, and $\angle A = 45°$.

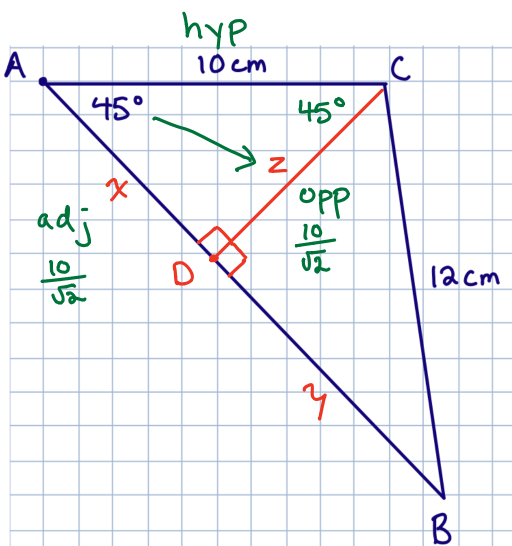
   a) Determine the exact length of side c.

   HINTS: → drop a perpendicular from $\angle C$ to side c.

   Consider adding something to the diagram so that you can use primary trig ratios.

   Think about using a proportion (see your answer to example 1 above).

   b) Determine the measure of $\angle C$, to the nearest whole degree.



Let x be the length of AD in cm
Let y be the length of DB in cm.
Let z be the length of DC in cm.

From example 1:

$$\cos 45° = \frac{1}{\sqrt{2}}$$

From this question:

$$\cos 45° = \frac{x}{10}$$

Set these equations equal to each other...

$$\frac{1}{\sqrt{2}} = \frac{x}{10}$$

$$10\left[\frac{1}{\sqrt{2}}\right] = \left[\frac{x}{\cancel{10}}\right]\cancel{10}$$

$$\frac{10}{\sqrt{2}} = x$$

By SATT, $\angle ACD = 45°$

Then $\triangle ACD$ is Isosceles.

By Isosceles T.T., $x = z$

$$\therefore z = \frac{10}{\sqrt{2}}$$

Now, use Pythagorean Theorem to find y.

$$12^2 = \left(\frac{10}{\sqrt{2}}\right)^2 + y^2$$

$$144 = \left(\frac{10}{\sqrt{2}}\right)\left(\frac{10}{\sqrt{2}}\right) + y^2$$

$$144 = \frac{100}{2} + y^2$$

$$144 = 50 + y^2$$

$$-50 \quad -50$$

$$94 = y^2$$

$$\sqrt{94} = \sqrt{y^2}$$

$$\sqrt{94} = y$$

Now, side c:

$$z + y$$

$$= \frac{10}{\sqrt{2}} + \sqrt{94}$$

c) $\sin B = \frac{opp}{hyp}$

$$= \frac{\frac{10}{\sqrt{2}}}{12}$$

$$\doteq 0.589$$

$$\angle B = \sin^{-1}(0.589)$$

$$\doteq 36°$$

By SATT $\angle DCB = 180 - 90° - 36$

$$= 54°$$

Finally... $\angle ACB$ or $\angle C = \angle DCA + \angle DCB$

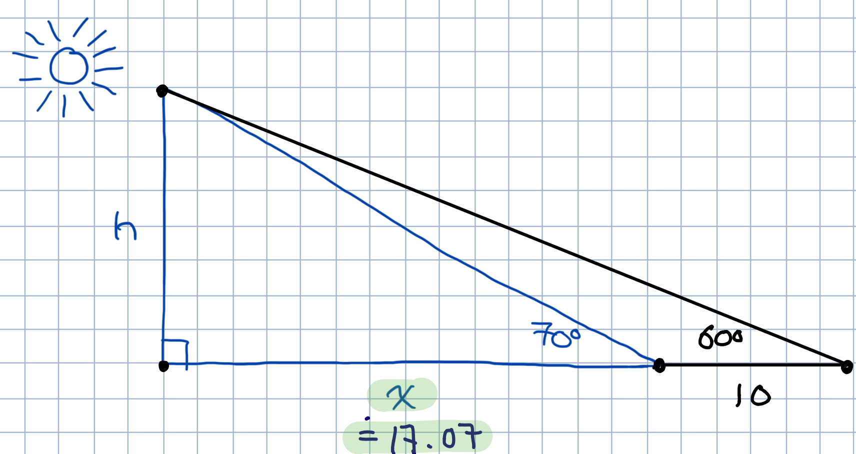
$$= 45 + 54$$

$$\doteq 99°$$

2. If the shadow of a building increases by 10 meters when the angle of elevation of the sun rays decreases from 70º to 60º, what is the height of the building?

   HINTS:

   The start of this solution is given below; add something to the diagram given.



Let h be the height of the building, in metres.

Let x be the initial length of the shadow, in metres.

$$\tan 70^\circ = \frac{h}{x} \qquad \tan 60^\circ = \frac{h}{x+10}$$

$$x[\tan 70^\circ] = \left[\frac{h}{\cancel{x}}\right]\cancel{x} \qquad (x+10)[\tan 60^\circ] = \left[\frac{h}{\cancel{x+10}}\right]\cancel{(x+10)}$$

$$x\cdot(\tan 70^\circ) = h \qquad (x+10)(\tan 60^\circ) = h$$

Set these equations equal to one another, then, isolate x.

$$x(\tan 70^\circ) = (x+10)(\tan 60^\circ)$$

$$x(\tan 70^\circ) = x(\tan 60^\circ) + 10(\tan 60^\circ)$$

$$-x(\tan 60^\circ) \qquad -x(\tan 60^\circ)$$

$$x(\tan 70^\circ) - x(\tan 60^\circ) = 10(\tan 60^\circ)$$

$$2.747x - 1.732x \doteq 17.321$$

$$1.015x = 17.321$$

$$\frac{1.015x}{1.015} = \frac{17.321}{1.015}$$

$$x \doteq 17.07$$

Now, we can obtain h.

$$\tan 70^\circ = \frac{h}{x}$$

$$\tan 70^\circ \doteq \frac{h}{17.07}$$

$$17.07[\tan 70^\circ] = \left[\frac{h}{\cancel{17.07}}\right]\cancel{17.07}$$

$$47 \doteq h$$

∴, the height of the building is about 47 m.