Solving Linear Systems by Substitution

Example 1 Find the point of intersection (that is, solve for x and y).

A $2x + y = 2$
B $3x - 2y = 10$

From **A**: $y = -2x + 2$

Substitute into **B**:

$$\begin{aligned} 3x - 2y &= 10 \\ 3x - 2(-2x + 2) &= 10 \\ 3x + 4x - 4 &= 10 \\ 7x - 4 &= 10 \\ 7x &= 10 + 4 \\ 7x &= 14 \\ x &= \frac{14}{7} \\ x &= 2 \end{aligned}$$

This is only half a solution. We need an ordered pair (x, y) -- the intersection point.

We have x, so how to find y?

Substitute $x = 2$ into **A**, the <u>other</u> equation**:**

$$\begin{aligned} 2x + y &= 2 \\ 2(2) + y &= 2 \\ 4 + y &= 2 \\ y &= 2 - 4 \\ y &= -2 \end{aligned}$$

∴ the solution, or point of intersection, is $(2, -2)$.

Example 2 Find the point of intersection (that is, solve for x and y).

A: $3x + 6y = 4$
B: $x - 2y = 1$

From **B** (since it has a term with a co-efficient of 1): $x = 1 + 2y$

Substitute into **A**:

$$\begin{aligned} 3x + 6y &= 4 \\ 3(1 + 2y) + 6y &= 4 \\ 3 + 6y + 6y &= 4 \\ 3 + 12y &= 4 \\ 12y &= 4 - 3 \\ 12y &= 1 \\ y &= \frac{1}{12} \end{aligned}$$

Leave your answer as a fraction in lowest terms, unless the decimal equivalent terminates.

Sub $y = \frac{1}{12}$ into the <u>other</u> equation, **B**:

$$\begin{aligned} x - 2y &= 1 \\ x - 2\left(\frac{1}{12}\right) &= 1 \\ x - \frac{2}{12} &= 1 \\ x - \frac{1}{6} &= 1 \\ x &= 1 + \frac{1}{6} \\ x &= \frac{6}{6} + \frac{1}{6} \\ x &= \frac{7}{6} \end{aligned}$$

Use improper fractions, instead of mixed fractions.

$\therefore$, the solution, or point of intersection, is $\left(\frac{7}{6}, \frac{1}{12}\right)$.

Example 3: Check the solution to the linear system.

By the method of substitution, you found that the solution to the system:

A: $x - y = 7$
B: $2x + y = -10$

… was: $(-1, -8)$

Check the solution. Must do a left-side / right-side check. **Form matters**!

Equation A

L.S.: $x - y$ R.S.: $= 7$

$= (-1) - (-8)$

$= -1 + 8$

$= 7$

Equation B

L.S.: $2x + y$ R.S.: $= -10$

$= 2(-1) + (-8)$

$= -2 - 8$

$= -10$

∵ LS = RS in both equations, $(-1, -8)$ is the sol'n.