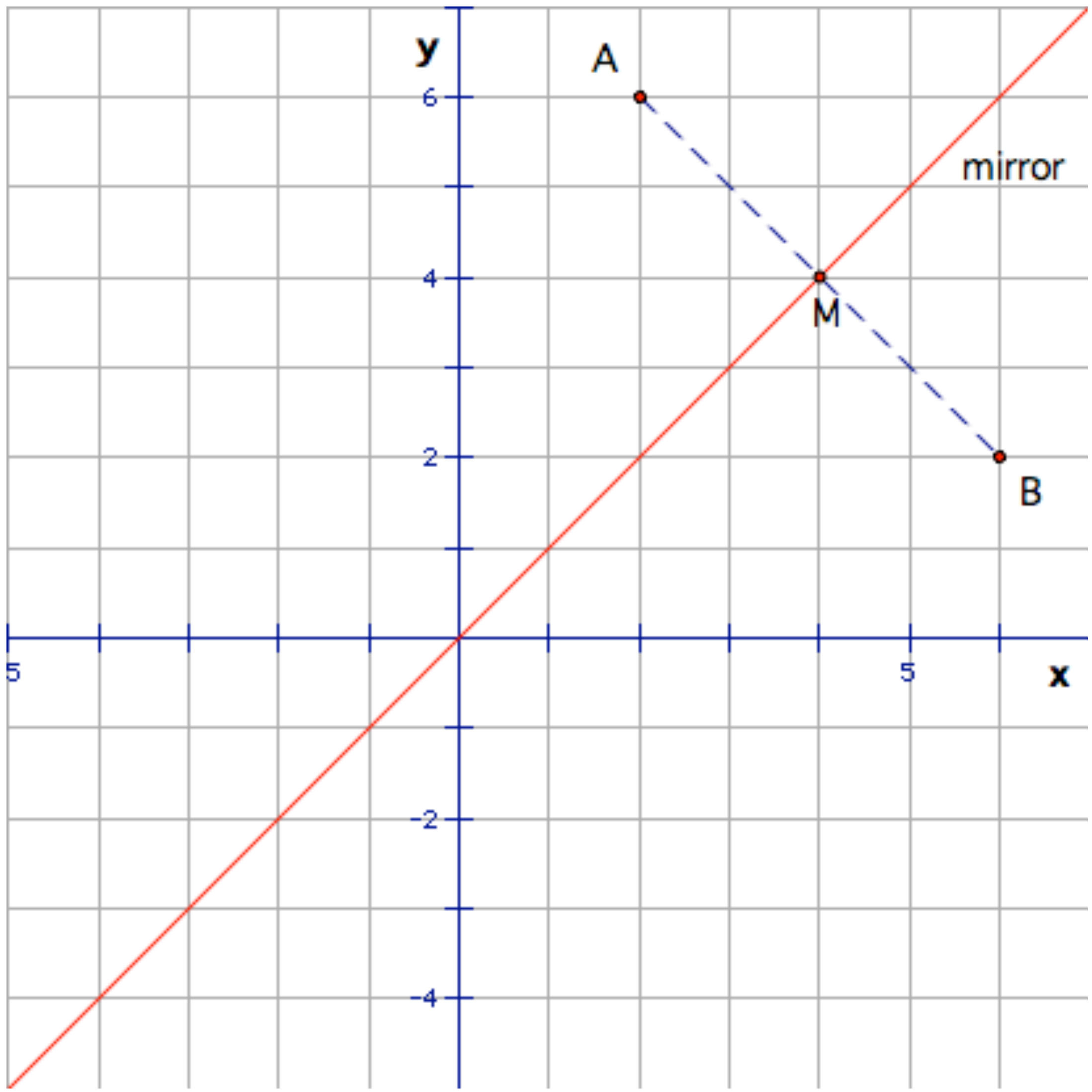
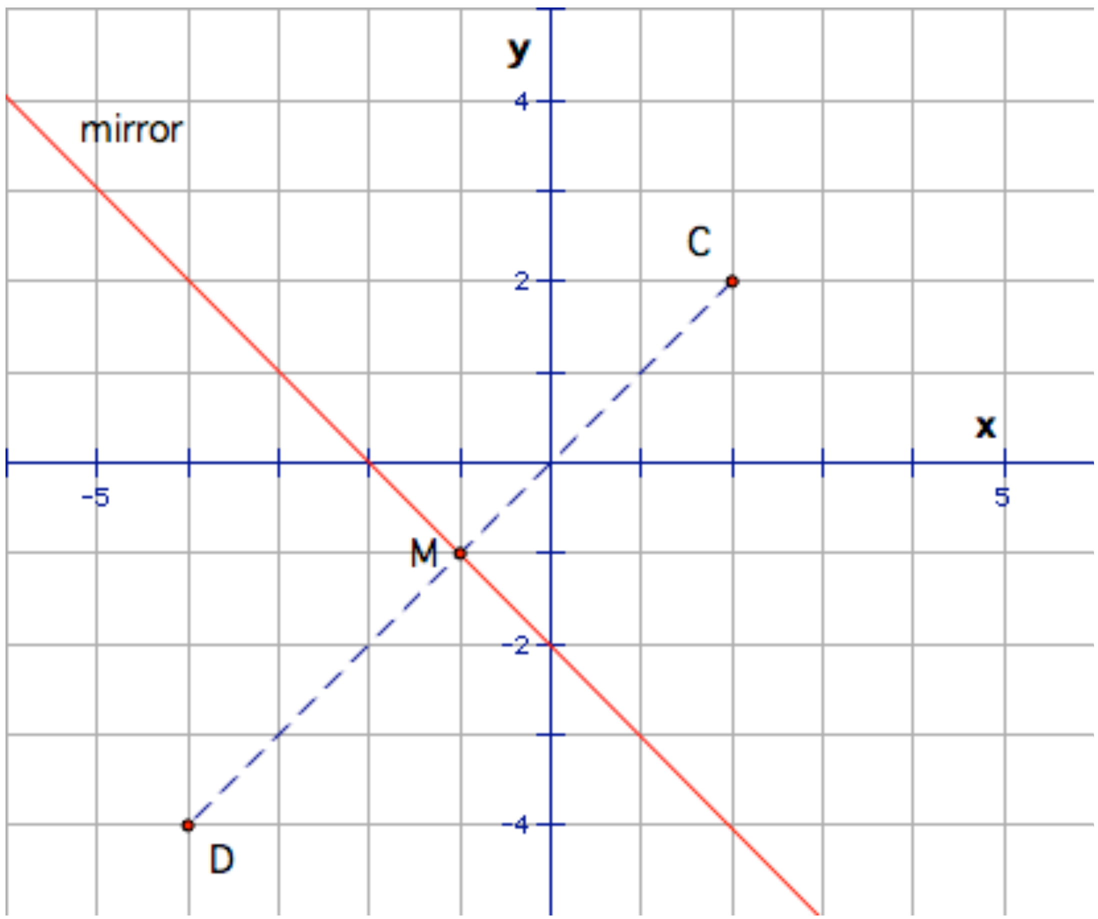
Midpoint of a Line Segment



	x	y
A	(2,	6)
B	(6,	2)
M	(4,	4)

M is the midpoint of AB.

Can think of B as the "reflection" of A.



	x	y
C	(2,	2)
D	(−4,	−4)
M	(−1,	−1)

M is the midpoint of CD.

Can think of D as the "reflection" of C.

In general... if A has co-ordinates (x_1, y_1) and B has co-ordinates (x_2, y_2) then the co-ordinates of the midpoint, M, are:

$$\boxed{\boxed{\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}}$$

remember this...

Ex. 1: Determine the co-ordinates of the midpoint, M, of the line segment with endpoints A(-2, -3) and B(4, 7).

x_1 y_1 x_2 y_2

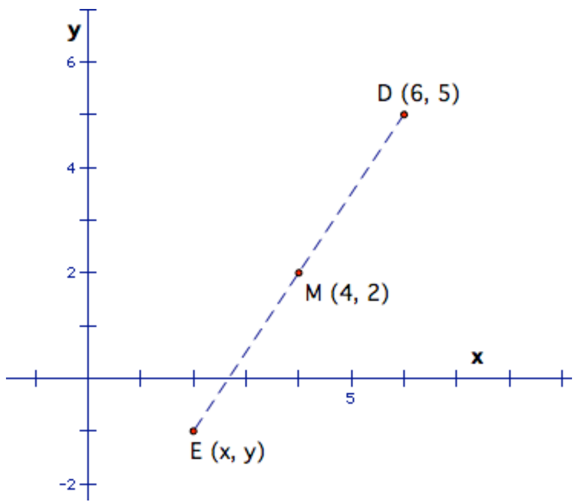
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-2 + 4}{2}, \frac{-3 + 7}{2} \right)$$

$$= \left(\frac{2}{2}, \frac{4}{2} \right)$$

$$= (1, 2)$$

Ex. 2: For a line segment DE, one endpoint is D(6, 5) and the midpoint is M(4, 2). Find the co-ordinates of endpoint E.



$$midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(4, 2) = \left(\frac{6 + x}{2}, \frac{5 + y}{2}\right)$$

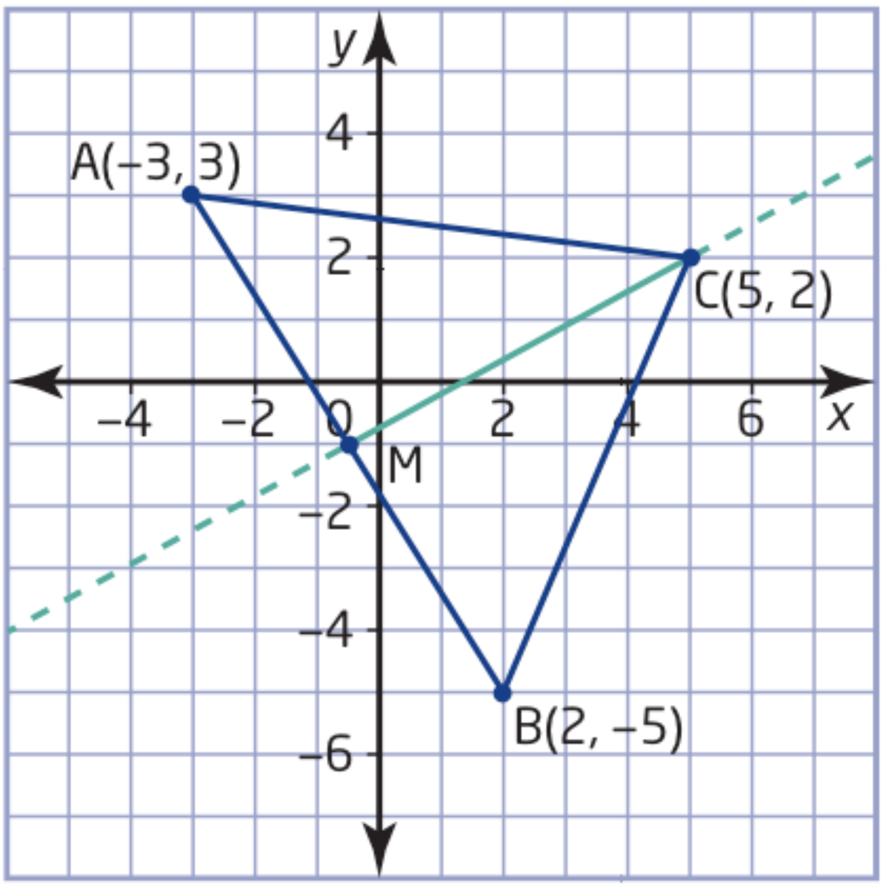
$$4 = \frac{6 + x}{2} \quad and \quad 2 = \frac{5 + y}{2}$$

$$x = 2 \qquad\qquad y = -1$$

$$E\,(2, -1)$$

Ex. 3

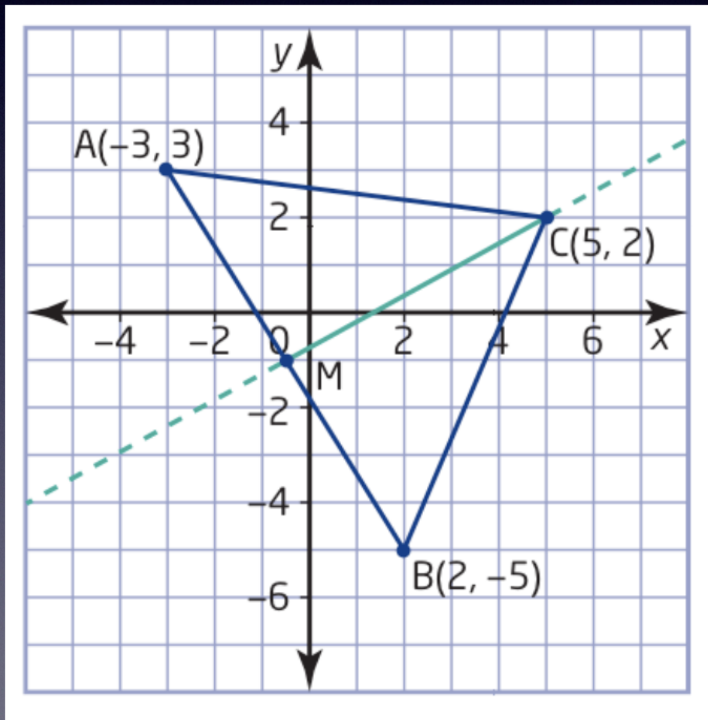
Determine an equation for the median from vertex C for the triangle with vertices C(5, 2), A(-3, 3), and B(2, -5).



HINT: If you are stuck on what a question is asking, consult your Geometry Glossary notes!

Let's work on this question in pairs at the whiteboard.

Determine an equation for the median from vertex C for the triangle with vertices C(5, 2), A(-3, 3), and B(2, -5).



HINT: If you are stuck on what a question is asking, consult your Geometry Glossary notes!

Let's work on this question in pairs at the whiteboard.

To get the equation of a line, we need the:

- slope
- y-intercept

$$y = mx + b$$

If we have two points on the line, we can find the slope using:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m = \frac{\Delta y}{\Delta x}$$

On the median line we have C(5, 2) but we don't know what the co-ordinates of M are.

We can find them, though, since M is the midpoint of $\overline{AB}$.

$$M_{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-3 + 2}{2}, \frac{3 + (-5)}{2}\right)$$

$$= \left(-\frac{1}{2}, -1\right)$$

Now get the slope of $\overline{MC}$: $M(-\frac{1}{2}, -1)$ $C(5, 2)$

x_1 y_1 x_2 y_2

$$m_{MC} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2-(-1)}{5-(-\frac{1}{2})}$$

$$= \frac{3}{5\frac{1}{2}}$$

$$= \frac{3}{\frac{11}{2}}$$

$$= 3\left(\frac{2}{11}\right)$$

$$= \frac{6}{11}$$

Finally, get y-intercept of $\overline{MC}$:

$$y = mx + b$$

$$y = \frac{6}{11}x + b$$

Sub in either point. $C(5, 2)$ may be easier.

x y

$$2 = \frac{6}{11}(5) + b$$

$$2 = \frac{30}{11} + b$$

$$-\frac{30}{11} \qquad -\frac{30}{11}$$

$$2 - \frac{30}{11} = b$$

$$\frac{22}{11} - \frac{30}{11} = b$$

$$-\frac{8}{11} = b$$

Whew! The equation of the median $\overline{MC}$ is $y = \frac{6}{11}x - \frac{8}{11}$.