Length of a Line Segment

Before you begin…

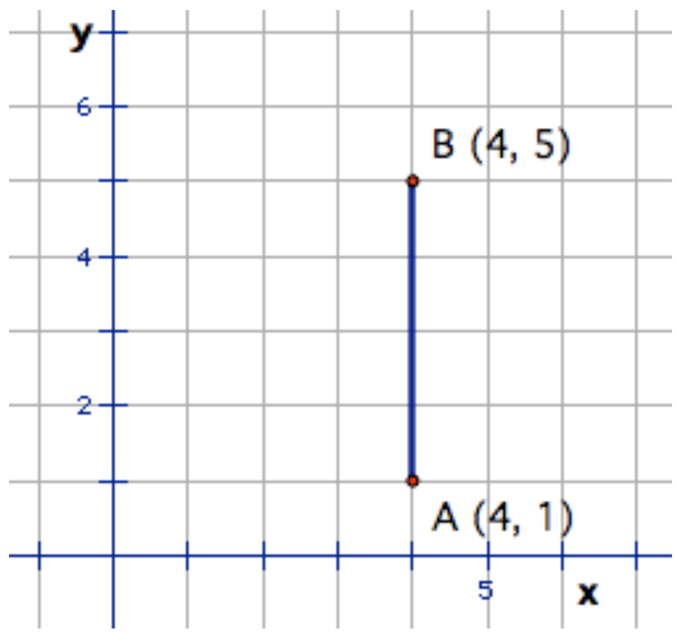
- This is a self-led slideshow.
- To advance, simply move to the next page in the PDF file.
- When asked to consider something before moving to the next slide, please take the time to stop and think.
- If you'd like to clarify something, by all means, please ask! 😀

Length of a Line Segment

- How can the formula to calculate the length of a line segment be developed?
- Far more easily than you think… and it will leverage an existing theorem that you already know well!
- Advance to the next slide to begin…

Can you guess the line segment length?

Think it over, then advance to reveal the answer.



Length of line segment AB is 4 units.

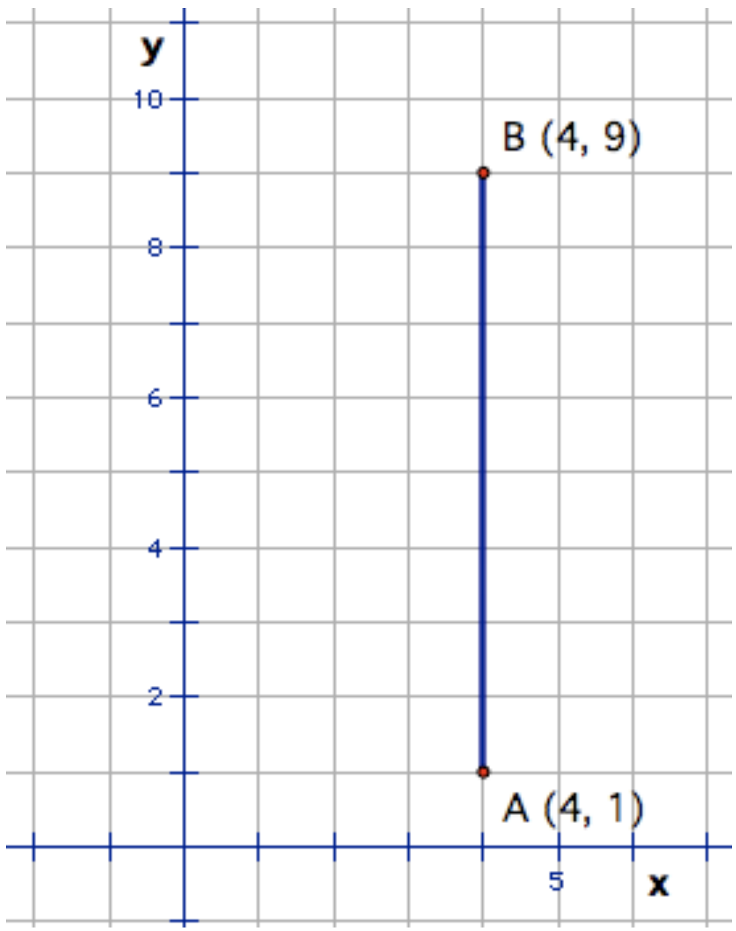
How did you figure out the line segment length?

Did you count squares on the grid?

Is there another way to determine the line segment length?

Can you guess the line segment length?

Think it over, then advance to reveal the answer.

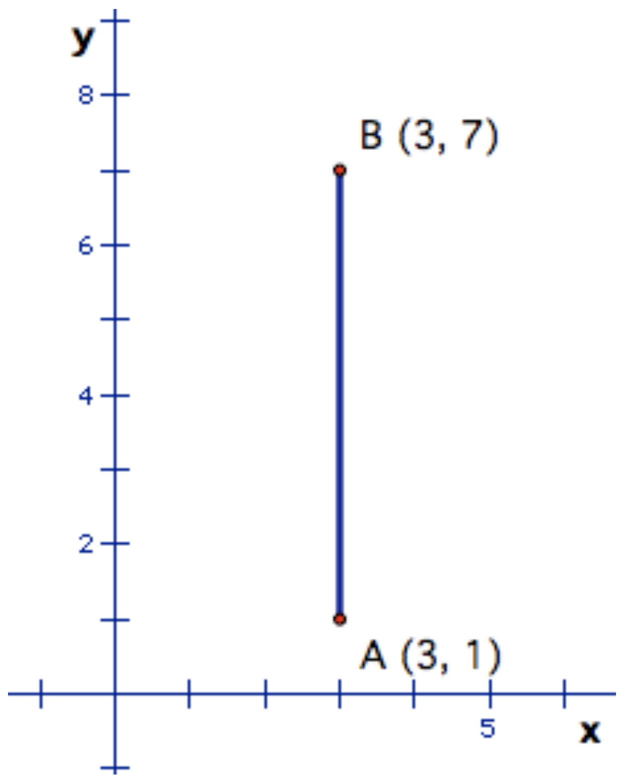


Length of line segment AB is 8 units.

Did you figure out another way to find the line segment length?

Can you guess the line segment length?

Think it over, then advance to reveal the answer.

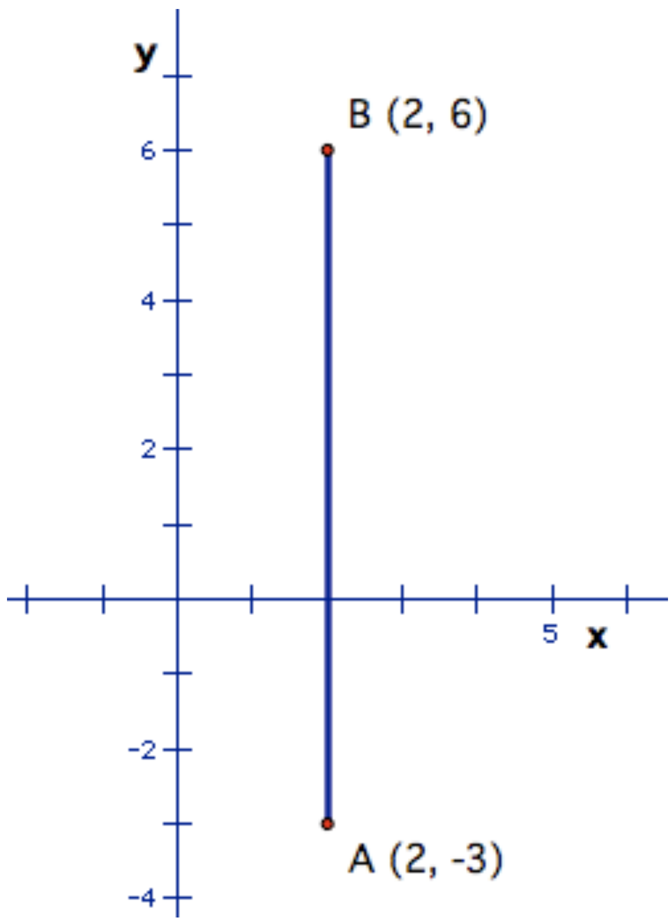


Length of line segment AB is 6 units.

Grab a sheet of paper, or open a file in a word processor or TextEdit. Write down your theory as to how to find the line segment length.

Can you guess the line segment length?

Think it over, then advance to reveal the answer.

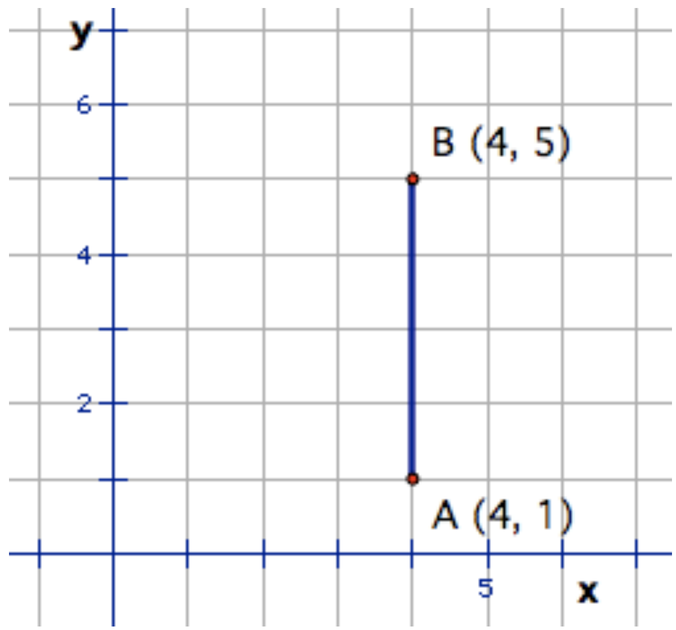


Length of line segment AB is 9 units.

You can find the length of a vertical line segment by subtracting the y-coordinates of each endpoint.

For example...

y_2 : y-coordinate of B is 5 y_1 : y-co-ordinate of A is 1

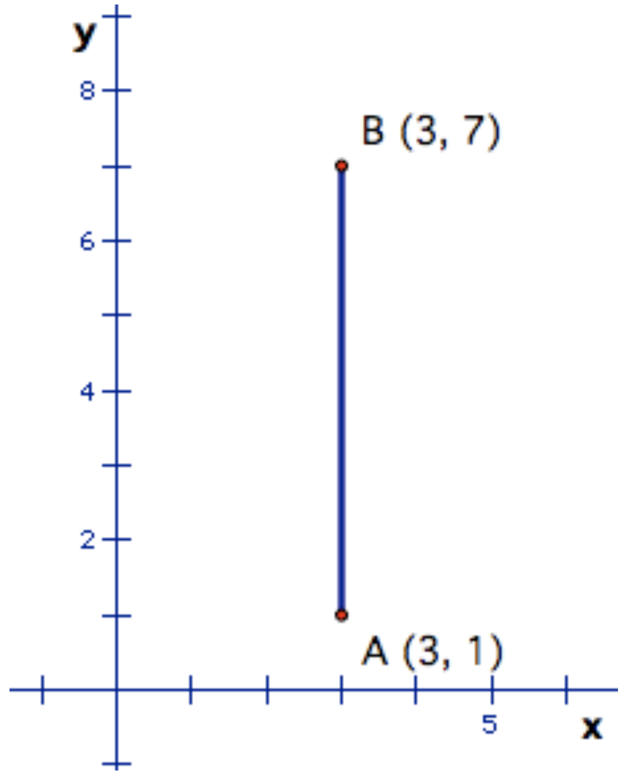


$$y_2 - y_1 = 5 - 1 = 4$$

So the length of the line segment is 4 units.

Another example...

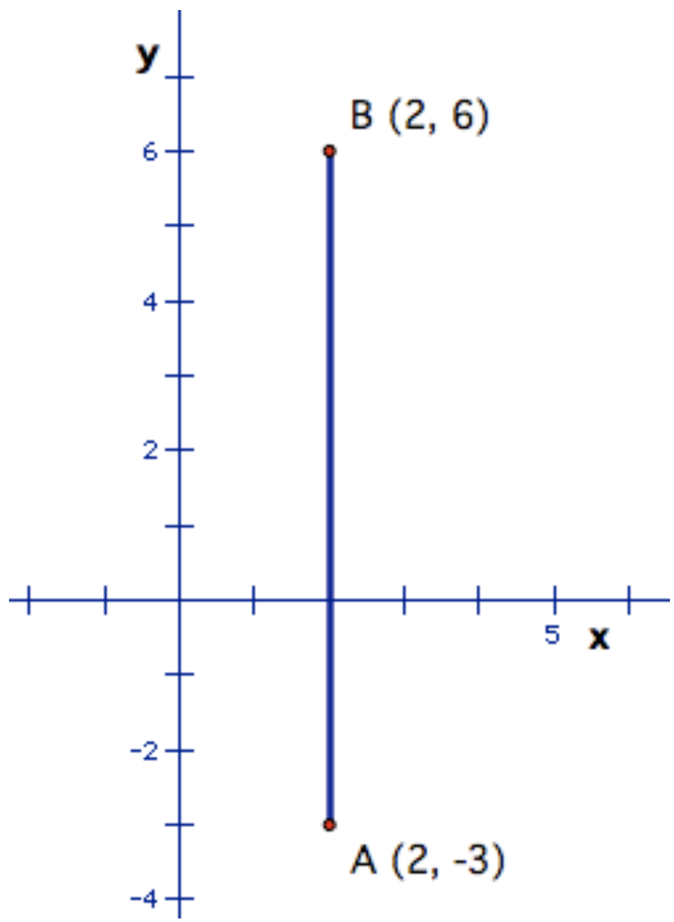
y_2 : y-coordinate of B is 7 y_1 : y-co-ordinate of A is 1



$$y_2 - y_1 = 7 - 1 = 6$$

So the length of the line segment is 6 units.

It even works when a y-coordinate is negative...



y_2 : y-coordinate of B is 6

y_1 : y-co-ordinate of A is -3

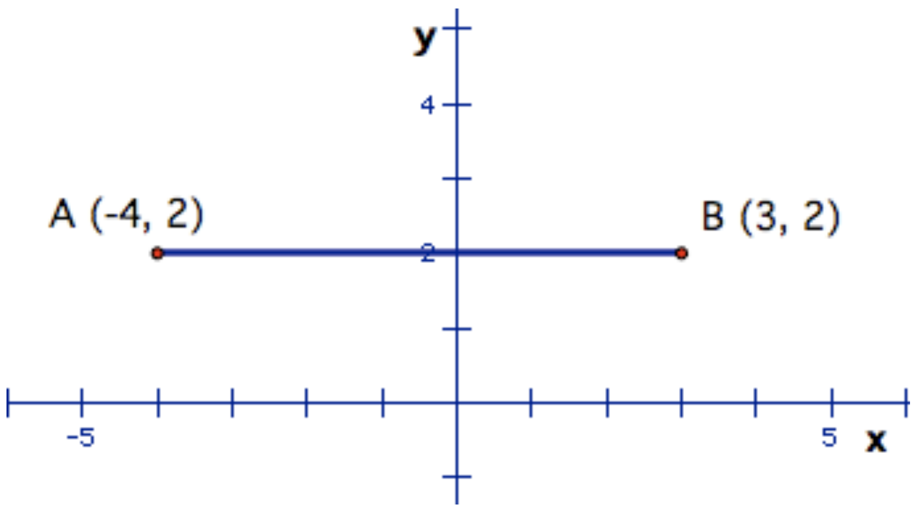
$y_2 - y_1 = 6 - (-3) = 6 + 3 = 9$

So the length of the line segment is 9 units.

Great! So we are set for finding the length of vertical line segments.

Will this work for horizontal line segments?

Let's take a look.



What do you think the length is?

The length is 7 units.

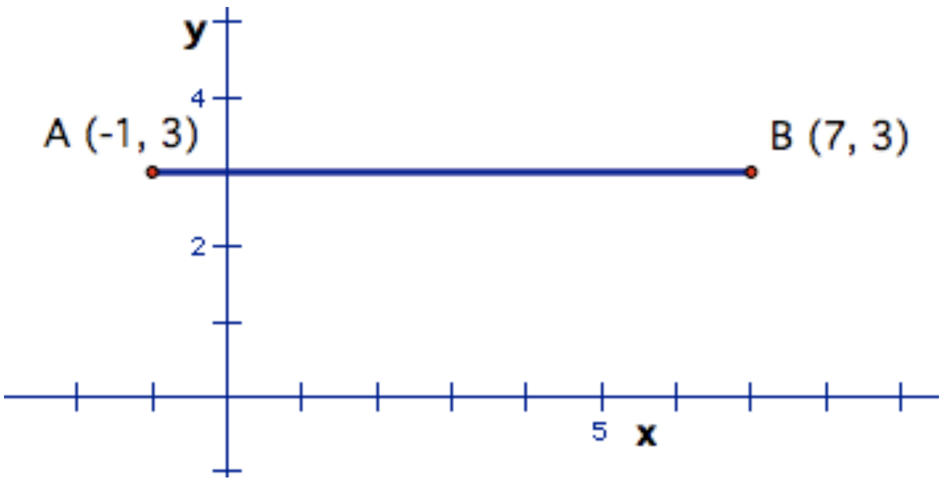
You can find the length the same way.
Just subtract the x-coordinates instead.

x_2 : x-coordinate of B is 3

$x_2 - x_1 = 3 - (-4) = 3 + 4 = 7$

x_1 : x-co-ordinate of A is -4

Let's try one more.



What do you think the length is?

The length is 8 units.

Subtract x-coordinates.

x_2 : x-coordinate of B is 7

x_1 : x-co-ordinate of A is -1

$x_2 - x_1 = 7 - (-1) = 7 + 1 = 8$

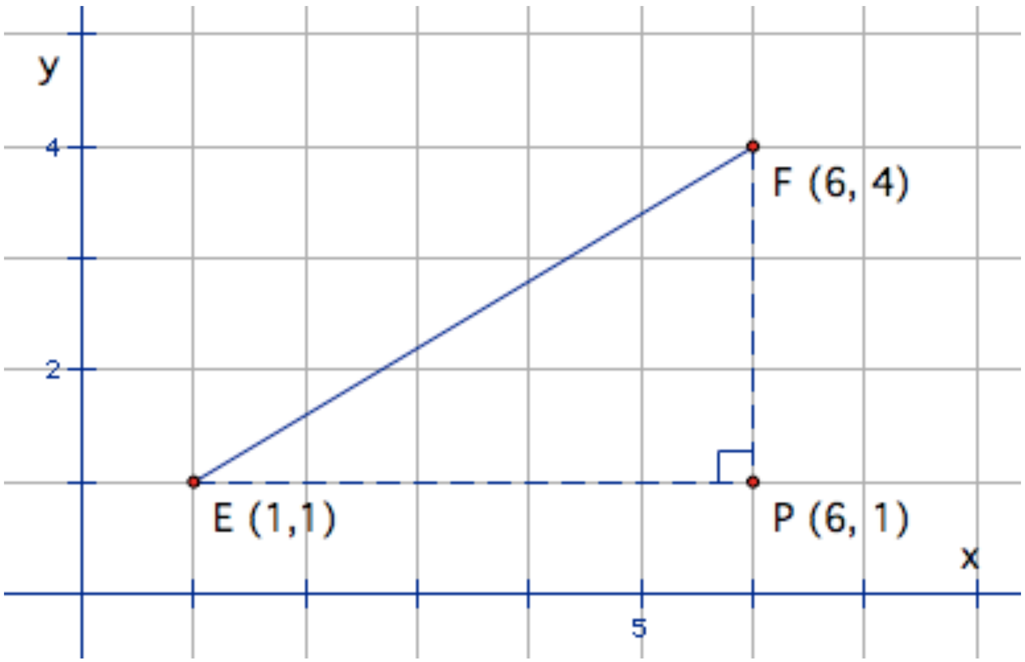
"Uh, sir?"

"*Yes?*"

"What if the line segment is slanted? Like, not horizontal or vertical?"

"*I thought you might ask that.*"

Hello, Pythagoras!



EF is slanted.

length of EF?

we can use Pythagorean Theorem

$$EF^2 = EP^2 + FP^2$$

$$EF^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

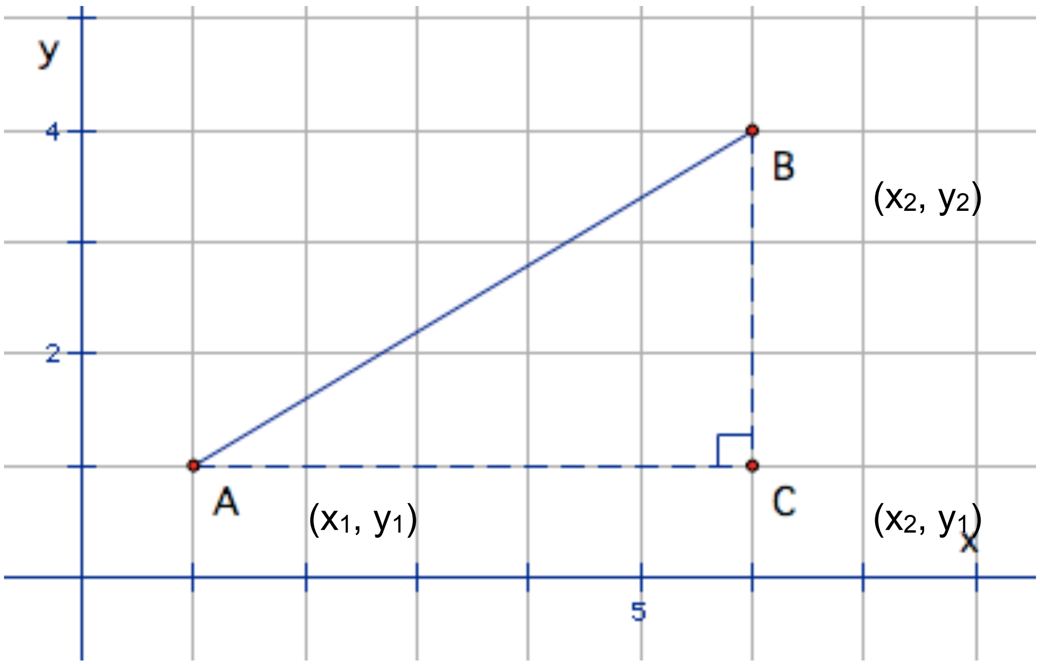
$$EF = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{5^2 + 3^2}$$

exact solution → $= \sqrt{34}$

approximate solution → $\cong 5.8$

In general... developing the formula for length of a line segment:



$$AB^2 = AC^2 + BC^2$$

$$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

or...

$$\ell = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

like, write this down!

Ex. 1: Find the length of the line segment joining the points $(2, -4)$ and $(-4, 7)$.

x_1 y_1 x_2 y_2

Provide an exact solution and an approximate solution to the nearest tenth.

$$\ell = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(-4 -2)^2 + (7 -(-4))^2}$$

$$= \sqrt{(-6)^2 + 11^2}$$

$$= \sqrt{36 + 121}$$

$$= \sqrt{157}$$

$$\cong 12.5$$

Ex. 2: A triangle has vertices A(-2, -5) B(-3, 2) and C(1,3).

a) Classify the triangle as equilateral, isosceles, or scalene.

b) Determine the perimeter of the triangle, to the nearest tenth of a unit.

How to solve, without graphing or tech?

- Draw a diagram.
- Understand terms (equilateral, isosceles, scalene).
- Find the length of each side of the triangle, then classify.
- To find perimeter, add the side lengths together.
- See a full solution here.

Full solution to final example from "Length of a Line Segment" lesson.

Ex 2. A(-2, -5) B(-3, 2) C(1, 3)

a)

$l_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2-(-3))^2 + (-5-2)^2}$

$= \sqrt{(-2+3)^2 + (-7)^2}$

$= \sqrt{(1)^2 + (-7)^2}$

$= \sqrt{1 + 49}$

$= \sqrt{50}$

use subscripts to indicate what line segment you are finding the length of!

when substituting a negative number, use brackets! eg. $-(-3) = +3$

better to leave answer as exact solution (in other words, do not change to a decimal and round off, which creates an approximate answer)

$l_{BC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-3-1)^2 + (2-3)^2}$

$= \sqrt{(-4)^2 + (-1)^2}$

$= \sqrt{16 + 1}$

$= \sqrt{17}$

$l_{AC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2-1)^2 + (-5-3)^2}$

$= \sqrt{(-3)^2 + (-8)^2}$

$= \sqrt{9 + 64}$

$= \sqrt{73}$

"therefore" → $\therefore$ ΔABC is scalene

$\because l_{AB} \neq l_{BC} \neq l_{AC}$ (all three side lengths are different)

"since" "is not equal to"

b) $P_{\Delta ABC} = l_{AB} + l_{BC} + l_{AC}$

$= \sqrt{50} + \sqrt{17} + \sqrt{73}$

$\doteq 7.071 + 4.123 + 8.544$

$\doteq 19.7$

the perimeter of triangle ABC

$\therefore$ the perimeter of ΔABC is approximately 19.7 units.

means answer on this line has been rounded (it is approximately equal to the line above)