

Factoring Trinomials

Factoring Trinomials where $a \neq 1$

A second-degree trinomial (that is, a quadratic) is written in the form $ax^2 + bx + c$.

You already know how to factor expressions where $a = 1$.

Try this one, where $a = 1$, $b = 15$, and $c = 50$:

$$\begin{aligned} & x^2 + 15x + 50 \\ & = (x+5)(x+10) \end{aligned}$$

How can we factor an expression like this one... where $a = 3$, $b = 8$ and $c = 4$?

$$\begin{aligned} & 3x^2 + 8x + 4 \\ & = 3x^2 + 2x + \underline{6x + 4} \\ & = x(\underline{3x+2}) + 2(\underline{3x+2}) \\ & = (3x+2)(x+2) \end{aligned}$$

① Get the product of "a" and "c"
 $3(4) = 12$

② List factors of 12
 $\{1, 12, 2, 6, 3, 4\}$ Look for factors that add to give "b"

③ Split ("decompose") middle term and then factor by grouping

Example 1

Factor, if possible.

$$\begin{aligned} a) & 2x^2 + 9x + 9 \\ & = 2x^2 + 3x + \underline{6x + 9} \\ & = x(\underline{2x+3}) + 3(\underline{2x+3}) \\ & = (2x+3)(x+3) \end{aligned}$$

$$2(9) = 18$$

Factors of 18

$$\begin{array}{l} 1, 18 \\ 2, 9 \\ 3, 6 \end{array}$$

$\{3, 6\}$ ← these factors add to give "b" value

Now, decompose and factor by grouping.

b) $3x^2 + 2x + 4$

not possible to factor!

$$3(4) = 12$$

Factors of 12

$$\begin{array}{l} 1,12 \\ 2,6 \\ 3,4 \end{array} \quad \left. \begin{array}{l} 1,12 \\ 2,6 \\ 3,4 \end{array} \right\}$$

none of these factors have

a sum of 2

∴, it is not possible to factor this expression.

c) $3x^2 + 7xy + 2y^2$

$$\begin{aligned} &= \underline{3x^2 + x y} + \underline{6x y + 2y^2} \\ &= x(\underline{3x+y}) + 2y(\underline{3x+y}) \\ &= (3x+y)(x+2y) \end{aligned}$$

$$3(2) = 6$$

Factors of 6

$$\begin{array}{l} 1,6 \\ 2,3 \end{array}$$

- Note that two variables doesn't change anything!
- Same process to factor.

d) $16x^2 + 26x - 12$ Hmm... common factor first! Then carry on...

$$\begin{aligned} &= 2[8x^2 + 13x - 6] \quad 8(-6) = -48 \\ &= 2[8x^2 - 3x + 16x - 6] \\ &= 2[x(\underline{8x-3}) + 2(\underline{8x-3})] \\ &= 2(8x-3)(x+2) \end{aligned}$$

Factors of -48

$$\begin{array}{l} -1, 48 \\ -2, 24 \\ -3, 16 \end{array}$$

Remember...

- ① Always LOOK to common factor first!
- ② Not every trinomial can be factored!

Opportunity to Learn

Use this IXL page to master the concepts we have discussed above. Earn a "Smart Score" of 90% or better. IMPORTANT: Write out your answers on paper, then type into IXL.

P.4 Factor quadratics with other leading coefficients