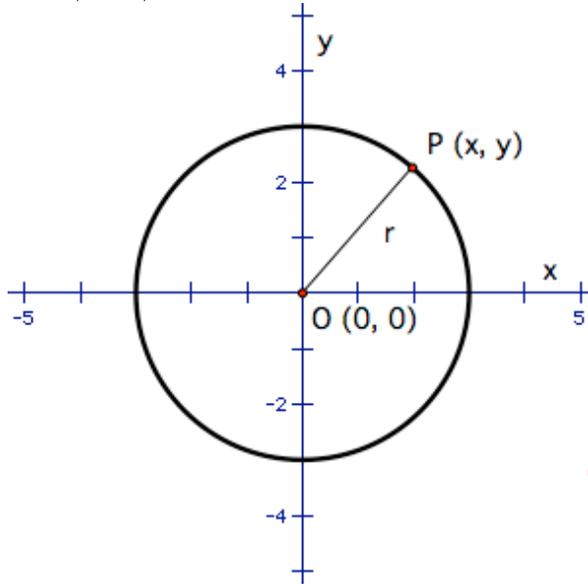## Equation of a Circle with Centre (0, 0)

Developing the equation…

Given the following circle with its centre at the origin $(0, 0)$ and a radius of r.

If $P(x, y)$ is any point on the circle...

The radius, r, is the line segment OP.

To find the length of r, what formula can we apply?

$$l_{OP} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{(x-0)^2 + (y-0)^2}$$

$$r = \sqrt{x^2 + y^2}$$

$$(r)^2 = \left(\sqrt{x^2 + y^2}\right)^2$$

$$r^2 = x^2 + y^2$$

So, the equation of a circle having a centre $(0, 0)$ is: $r^2 = x^2 + y^2$ or $x^2 + y^2 = r^2$

Example 1

Write an equation for the circle with centre $(0, 0)$ and radius of 3.

OK… any circle with centre $(0, 0)$ has the equation: $x^2 + y^2 = r^2$

Substitute $r = 3$…

$$x^2 + y^2 = (3)^2$$
$$x^2 + y^2 = 9$$

∴ the equation of a circle with centre $(0, 0)$ and radius 3 is: $x^2 + y^2 = 9$

Example 2

Find an equation for the circle that is centred at the origin and passes through the point $(4, 3)$.

OK… any circle at the origin, that is, having a centre at $(0, 0)$ has the equation:

$$x^2 + y^2 = r^2$$

Substitute the… *x and y values to find what r^2 equals.*

$$(4)^2 + (3)^2 = r^2$$
$$16 + 9 = r^2$$
$$25 = r^2$$

Now, substitute again, this time… r^2 … *so… the equation is*

$$x^2 + y^2 = 25$$

∴ the equation for the circle centred at $(0, 0)$ and passing through $(4, 3)$ is:
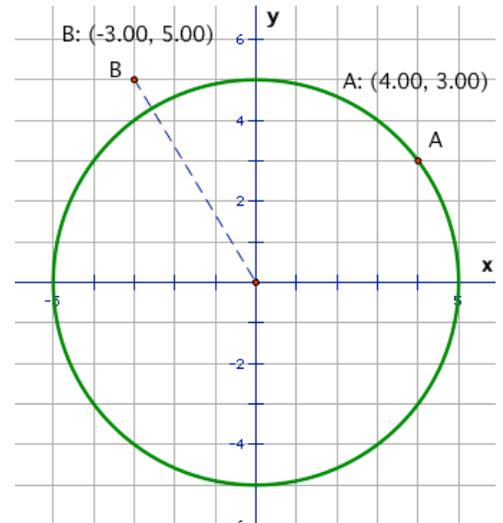
$$x^2 + y^2 = 25$$

Example 3

Use algebra to determine if the point $(-3, 5)$ is inside the circle described in example 2.

OK, the equation of the circle from the last example is: $x^2 + y^2 = 25$

Let's pretend $(-3, 5)$ is a point on the circumference of another circle. $\hookrightarrow r^2 = 25$

$r = 5$

If that's so, let's find the radius of that other circle…

$$x^2 + y^2 = r^2$$
$$(-3)^2 + (5)^2 = r^2$$
$$9 + 25 = r^2$$
$$34 = r^2$$
$$\sqrt{34} = \sqrt{r^2}$$
$$\sqrt{34} = r$$
$$5.8 \doteq r$$

∴ the radius of the circle from example 2 is 5 units, and the radius of a circle passing through $(-3, 5)$ is about 5.8 units, it must mean that $(-3, 5)$ lays outside the circumference of the circle from example 2.