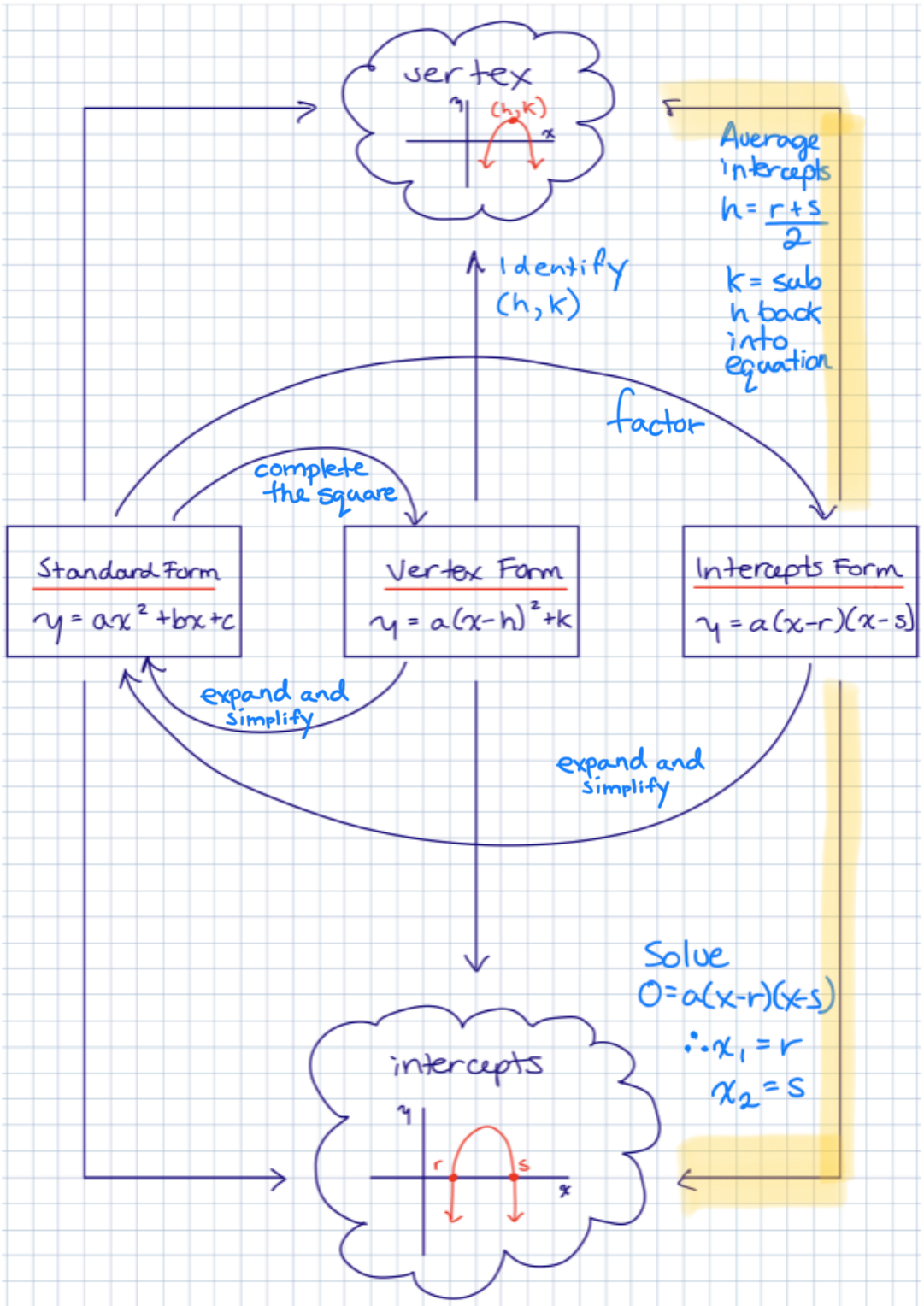
Graphing Quadratics Using the Intercepts and Vertex

Quadratic Relations Concept Map



Recall

Earlier this semester we learned how to graph quadratic relations given in the form $y = a(x - r)(x - s)$. The r and s values represent the x-intercepts – where the parabola crosses the x-axis.

Try graphing the following relation: $y = (x - 3)(x + 1)$

1. x-intercepts are 3 and -1
2. average intercepts to get x-value of vertex

$h = \frac{r+s}{2}$

$h = \frac{3+(-1)}{2}$

$= \frac{2}{2}$

$= 1$

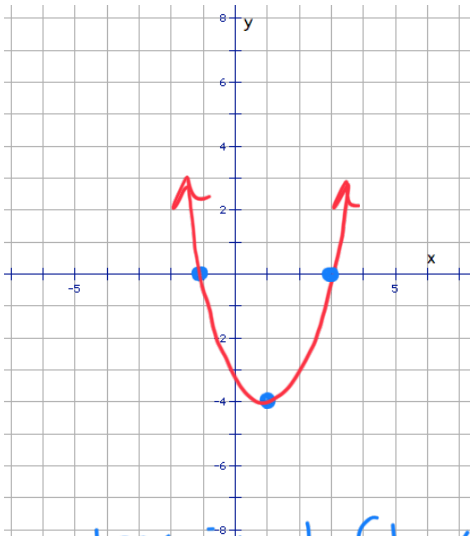
3. now substitute to get y-value of vertex:

$y = (x-3)(x+1)$

$k = (1-3)(1+1)$

$= (-2)(2)$

$= -4$



∴ vertex is at $(1, -4)$ (h, k)

Example 1

Graph each relation by factoring (do not complete the square).

a) $y = x^2 + 8x + 7$

$y = (x+7)(x+1)$

∴ x-ints are -7 and -1

$h = \frac{-7+(-1)}{2}$

$= \frac{-8}{2}$

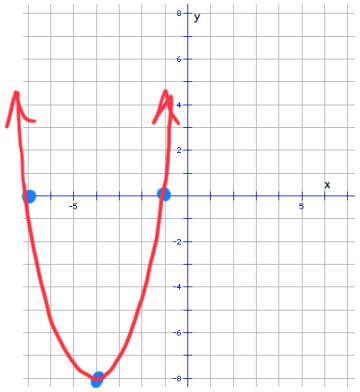
$= -4$

$k = (x+7)(x+1)$

$= (-4+7)(-4+1)$

$= (3)(-3)$

$= -9$



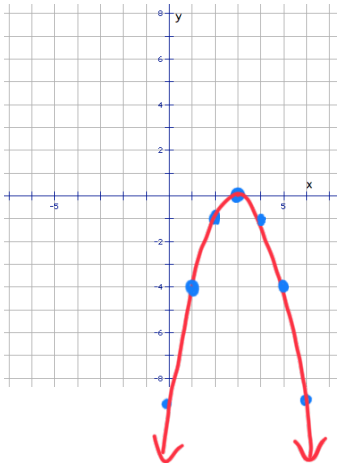
b) $y = -x^2 + 6x - 9$

$y = -(x^2 - 6x + 9)$

$y = -(x-3)(x-3)$

∴ x-int is 3

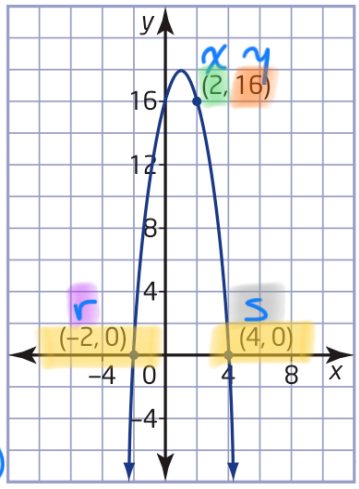
* only one intercept!
* this means vertex is on x-axis



* need more points... hmm...
* note "a" value is -1
* so parabola opens down with regular step pattern!

Example 2

Find the equation of the relation from the information given in the graph.



* Hmm... no vertex given. Cannot use $y = a(x-h)^2 + k$ form.

* Intercepts given. Use $y = a(x-r)(x-s)$ form!

$y = a(x-r)(x-s)$

Sub what we know! $16 = a(2-(-2))(2-4)$

$16 = a(2+2)(-2)$

$16 = a(4)(-2)$

$16 = a(-8)$

$\frac{16}{-8} = \frac{a(-8)}{-8}$

$-2 = a$

∴ equation is $y = -2(x+2)(x-4)$

Example 3

To commemorate the 100th anniversary of the Newtonville Fair, an entrance arch will be built. The design engineer uses the equation $h = -d^2 + 16$ to model the arch, where h is the height, in metres, above the ground and d is the horizontal distance, in metres, from the centre of the arch.

a) How wide and how tall is the arch?
b) For what values of d is the relation valid? Explain.
c) If a width of 2.5 m is needed per line-up at the entrance, how many line-ups can there be?

a) the arch is 16 metres tall (given by "k" value in equation).

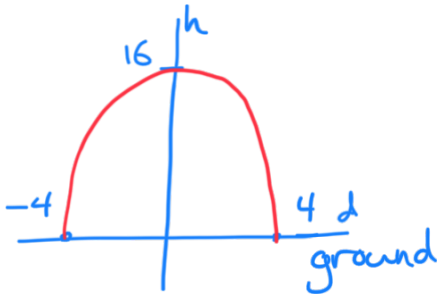
$h = -d^2 + 16$

$\frac{0}{-1} = \frac{-d^2 + 16}{-1}$

$0 = d^2 - 16$

$0 = (d-4)(d+4)$

∴ intercepts are 4 and −4



b) $-4 \le d \le 4$ (between −4 and 4... outside this span the arch does not exist)

c) 2.5 m per line...

... 2.5 + 2.5 + 2.5 = 7.5...

...so 3 line ups will fit!

Opportunity to Learn

Complete all questions in the handout that accompanies this lesson.

eg