

Finding the Equation of a Line

Linear models can be useful for making predictions in many situations involving direct and partial variation. If you know two points of information, you can find an equation for the line.

Step 1. Find the slope. Substitute the two known points into the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Step 2. Find the y-intercept. Substitute the slope and one of the two points into $y = mx + b$. Solve for b .

Step 3. Write the equation. Substitute the slope and y-intercept into $y = mx + b$.

Example Find an Equation for a Line, Given Two Points

- a) A line passes through (1, 2) and (5, 10). Find an equation for the line.
- b) A line passes through (-3, -2) and (6, -8). Find an equation for the line.

Solution

- a) *Step 1.* Find the slope.

Substitute (1, 2) and (5, 10) into the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

$$\begin{aligned} m &= \frac{10 - 2}{5 - 1} \\ &= \frac{8}{4} \text{ or } 2 \end{aligned}$$

The slope is 2.

Step 2. Find the y-intercept.

Substitute $m = 2$ and one of the points, say (1, 2), into $y = mx + b$.

$$2 = 2(1) + b$$

$$2 = 2 + b$$

$$0 = b$$

The y-intercept is 0.

Step 3. Write the equation.

Substitute $m = 2$ and $b = 0$ into $y = mx + b$.

$$\begin{aligned}y &= (2)x + (0) \\ &= 2x + 0 \\ &= 2x\end{aligned}$$

The equation of the line passing through (1, 2) and (5, 10) is $y = 2x$.

b) *Step 1.* Find the slope.

Substitute $(-3, -2)$ and $(6, -8)$ into the slope formula.

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-8 - (-2)}{6 - (-3)} \\ &= \frac{-8 + 2}{6 + 3} && \text{Simplify integer calculations.} \\ &= \frac{-6}{9} \\ &= -\frac{2}{3}\end{aligned}$$

The slope is $-\frac{2}{3}$.

Step 2. Find the y -intercept.

Substitute $m = -\frac{2}{3}$ and one of the points, say $(6, -8)$, into

$$y = mx + b.$$

$$-8 = \left(-\frac{2}{3}\right)(6) + b$$

$$-8 = -4 + b$$

$$-8 + 4 = b$$

$$-4 = b$$

The y -intercept is -4 .

Step 3. Write the equation.

$$y = mx + b$$

$$y = -\frac{2}{3}x + (-4) \quad \text{Substitute } m = -\frac{2}{3} \text{ and } b = -4.$$

The equation of the line is $y = -\frac{2}{3}x - 4$.