Factoring Quadratic Expressions: Inquiry, Application, Communication Questions

Complete each of the following questions on a separate sheet of lined paper.

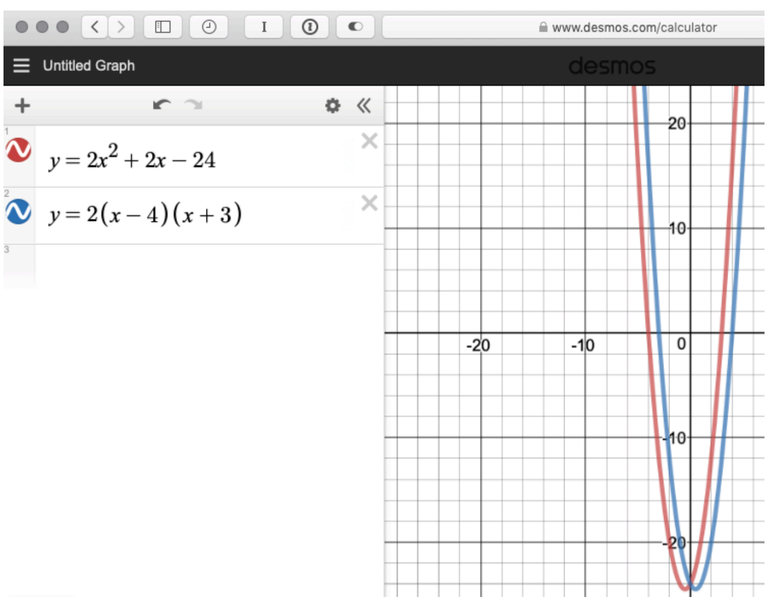
We will take these up in our next class together.

1. Steven tells Hudson that he's converted the equation of a quadratic relation from its standard form expression, $y = 2x^2 + 2x - 24$, to its equivalent expression in intercepts form, $y = 2(x - 4)(x + 3)$.

   Hudson is not so sure that Steven has done this correctly.

   Describe three different ways that Hudson could check to see whether Steven's work is correct. Include examples with each method you describe.

(A) Graph each relation.
The original expression and the expression in factored form will produce the same graph if they are truly equivalent expressions.



Using Desmos, we can see that the expressions actually are not equivalent.

(B) Substitute a value, like $x = 10$.
If the y-value you obtain is different from each expression, they are not equivalent.

(C) You could factor the original expression and see if you obtain what Scott got.

$y = 2x^2 + 2x - 24$
$y = 2(x^2 + x - 12)$
$y = 2(x + 4)(x - 3)$

So, Scott made a small error (swapped the + and - in the binomials).

2. The Sydney Harbour Bridge in Australia has an unusually wide and long-span surface.

   The bridge carries two rail lines, eight road lanes, a cycle lane, and a walkway.

   a. Factor the expression $10x^2 - 7x - 3$ to find binomials that represent the dimensions (length and width) of the rectangular surface of the bridge.

   b. If x represents 50 m, what is the actual length and width of the bridge, in metres?

a) $10x^2 - 7x - 3$

$= 10x^2 - 10x + 3x - 3$

$= 10x(x-1) + 3(x-1)$

$= (x-1)(10x+3)$

b) $A = \text{width}(\text{length})$

$= (x-1)(10x+3)$

$x = 50$

width

$x-1$

$50-1$

$= 49\text{m}$

length

$10x+3$

$= 10(50)+3$

$= 500+3$

$= 503\text{m}$

3. A parabola has the equation $y = x^2 + 2x - 15$.

   a. Identify the x -intercepts of the parabola.

   NOTE: If you are not sure how to do this, look at the quadratic relations concept map we have referred to several times in this unit of study.

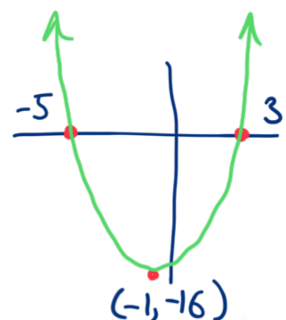
   b. Find the equation of the axis of symmetry.
   c. Find the vertex.
   d. Make a sketch of the graph.

a) $y = x^2 + 2x - 15$
$= (x+5)(x-3)$
$\therefore$ x-intercepts are -5 and 3

b) $x = -1$

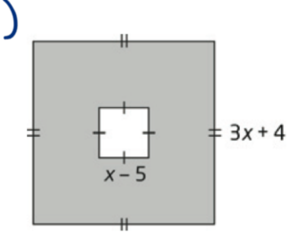
c) Sub $x = -1$ to get y-value
$y = x^2 + 2x - 15$
$= (-1)^2 + 2(-1) - 15$
$= 1 - 2 - 15$
$= -16$

d)



(sketch is not to scale)

4. For the figure shown at right:



   a. Find an algebraic expression for the area of the shaded region.
   b. Write the area expression in factored form.

a) $A_{shaded} = A_{large} - A_{small}$
$= (3x+4)^2 - (x-5)^2$
$= 9x^2 + 24x + 16 - (x^2 - 10x + 25)$
$= 9x^2 + 24x + 16 - x^2 + 10x - 25$
$= 8x^2 + 34x - 9$

b) $8x^2 + 34x - 9$ $\quad$ -72
$= 8x^2 - 2x + 36x - 9$ $\quad$ $-1, 72$
$= 2x(4x-1) + 9(4x-1)$ $\quad$ $-2, 36$
$= (4x-1)(2x+9)$

5. For each expression, find two possible values for k so that the trinomial is factorable over the integers.

   a. $x^2 + kx + 16$
   b. $3x^2 + kx + 25$
   c. $12x^2 + 11x + k$

a) $x^2 + kx + 16$

eg. $x^2 + 17x + 16$

or $x^2 + 10x + 16$

16

1, 16 = 17

2, 8 = 10

b) $3x^2 + kx + 25$

eg. $3x^2 + 76x + 25$

or $3x^2 + 28x + 25$

75

1, 75 = 76

3, 25 = 28

need these products... so find factors to obtain possible values for k

c) $12x^2 + 11x + k$

eg. $12x^2 + 11x - 1$

$12x^2 + 11x + 2$

Need k to provide a product that has factors with a sum of 11.

If k = −1, then −12

−1, 12 ✓

if k = 1, then 12

1, 12
2, 6 nope
3, 4

If k = 2 then 24

1, 24
2, 12
3, 8 ✓

6. State two possible trinomials that would have $(x - 7)$ as a factor.

   HINT: Remember that factoring is simply the opposite of multiplying. What creates a trinomial? (the product of two binomials)

Need $(x-7)$ as one factor. Pick any other binomial, and multiply to get a possible trinomial.

$(x-7)(x+1) = x^2 - 6x - 7$ or $(x-7)(x+2) = x^2 - 5x - 14$

7. Write two example expressions that fit each description:

   a. a second-degree trinomial

   $x^2 + x + 1$ or $xy + x + 1$

   b. a third-degree binomial that includes a constant term

   $x^3 + 1$ or $xyz + 2$

   c. the sum of a fourth-degree monomial and any third-degree trinomial

   $x^4 + (x^3 + x^2 + x)$ or $x^2y^2 + (xyz + yz + 2)$

   d. the difference of a constant term and a second-degree monomial

   $7 - x^2$ or $5 - xy$

8. How can you tell whether the right side of the relation $y = x^2 + 17x + 24$ is factorable over the integers? If it is not factorable over the integers, what does this mean about the graph of the relation? Do not actually factor the expression… instead, use point-form sentences to describe how you would determine if the expression can be factored.

   1. Look for factors of 24.
   2. Check the sums of those factors.
   3. If any sums of the factors equal 17, this means the expression can be factored over the integers.
   4. If the expression can be factored over the integers, it means the graph of the relation crosses the x-axis at integer values.

   eg.