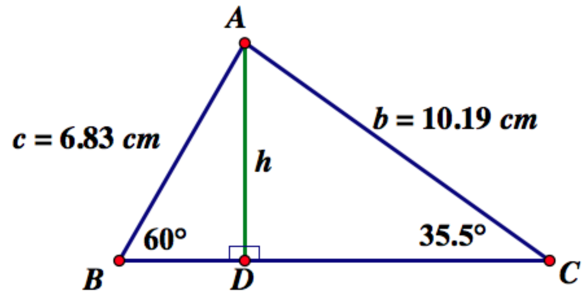
Developing the Sine Law

Example 1



a. What is the length of h, accurate to a tenth of a centimetre?

$$\sin B = \frac{opp}{hyp}$$

$$\sin 60° = \frac{h}{6.83}$$

$$6.83[\sin 60°] = \left[\frac{h}{\cancel{6.83}}\right]\cancel{6.83}$$

$$6.9 \doteq h$$

b. Confirm your response to a) another way.

$$\sin C = \frac{opp}{hyp}$$

$$\sin 35.5 = \frac{h}{10.19}$$

$$10.19[\sin 35.5] = \left[\frac{h}{\cancel{10.19}}\right]\cancel{10.19}$$

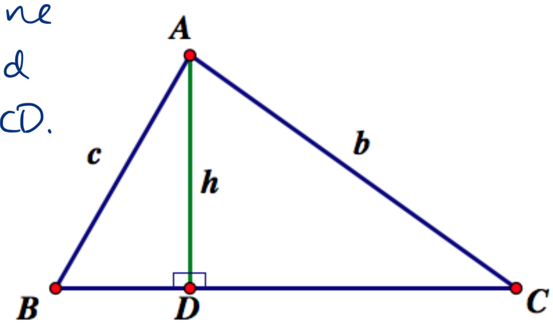
$$5.9 \doteq h$$

c. h is an altitude of triangle ABC.

Will h, calculated via the two smaller triangles, always be the same length, in any triangle with an altitude drawn like this? Why?

Yes. It is literally the same line segment, AD, in both $\triangle ABD$ and $\triangle ACD$.

Now express h in terms of the sides and angles of the triangle at right. Can you do this in two different ways?

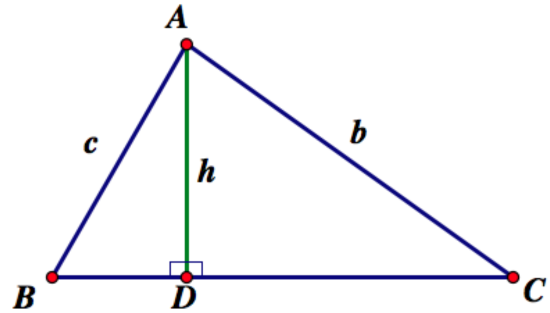


$$\sin B = \frac{opp}{hyp} \qquad \sin C = \frac{opp}{hyp}$$

$$\sin B = \frac{h}{c} \qquad \sin C = \frac{h}{b}$$

d. Given your answer to part c), can you create an equation that relates (connects) the lengths of sides b and c?

In other words, by the end of the solution, fill in the blanks below (but show how you got there).



$\sin B = \frac{h}{c}$ $\qquad$ $\sin C = \frac{h}{b}$

$c\left[\sin B\right] = \left[\frac{h}{\cancel{c}}\right]\cancel{c}$ $\qquad$ $b\left[\sin C\right] = \left[\frac{h}{\cancel{b}}\right]\cancel{b}$

$c(\sin B) = \boxed{h}$ $\qquad$ $b(\sin C) = \boxed{h}$

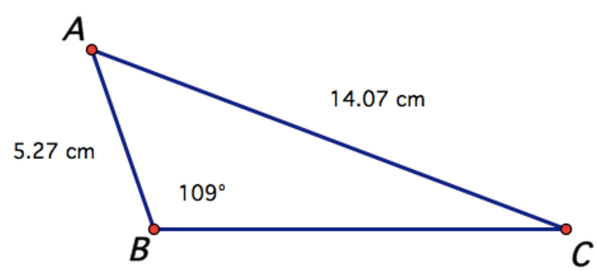
Set the two equations equal to each other!

$c(\sin B) = b(\sin C)$

$\frac{\cancel{c}(\sin B)}{b\cancel{c}} = \frac{\cancel{b}(\sin C)}{\cancel{b}c}$

$\frac{\sin B}{b} = \frac{\sin C}{c}$

e. Now let's say you had a triangle with measurements as shown, and needed to determine the measure of $\angle C$.



$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 109^\circ}{14.07} = \frac{\sin C}{5.27}$

$5.27\left[\frac{\sin 109^\circ}{14.07}\right] = \left[\frac{\sin C}{\cancel{5.27}}\right]\cancel{5.27}$

$0.354 \doteq \sin C$

$\sin^{-1}(0.354) = \angle C$

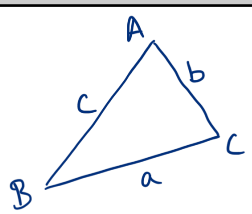
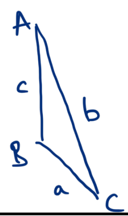
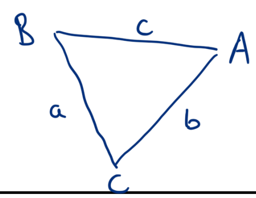
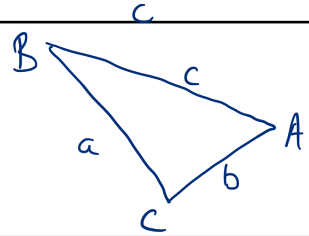
$21^\circ \doteq \angle C$

Example 2

Open this Geogebra sketch (www.geogebra.org/classic/awtb9wx3).

In the file, drag vertices A, B, and C, to create four very different looking triangles.

Then fill in the chart below, using the measurements shown in the file.

△	Rough sketch of the triangle	$\frac{a}{sin\,(A)}$	$\frac{b}{sin\,(B)}$	$\frac{c}{sin\,(C)}$
1		3.294	3.294	3.294
2		14.096	14.096	14.096
3		3.28	3.28	3.28
4		6.238	6.238	6.238

a. What do you notice about all the ratios for a given triangle?

For a given triangle, the three ratios are always equal.

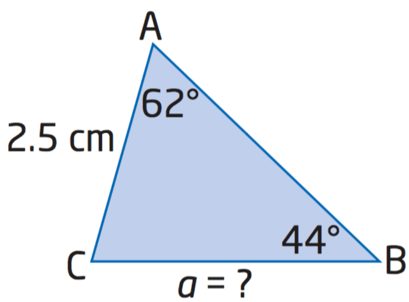
b. Write an equation that summarizes the relationship between these ratios for any given triangle.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

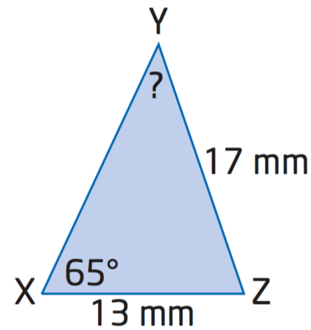
Example 3

For each triangle, find the measure of the indicated angle or side.

a.



b.



a) $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{\sin 62°} = \frac{2.5}{\sin 44°}$

$\cancel{\sin 62°}\left[\frac{a}{\cancel{\sin 62°}}\right] = \left[\frac{2.5}{\sin 44}\right]\sin 62°$

$a = 3.1776\ldots$

$a \doteq 3.2$

b) $\frac{\sin Y}{Y} = \frac{\sin X}{X}$

$\frac{\sin Y}{13} = \frac{\sin 65°}{17}$

$\cancel{13}\left[\frac{\sin Y}{\cancel{13}}\right] = \left[\frac{\sin 65°}{17}\right]13$

$\sin Y \doteq 0.693$

$\angle Y = \sin^{-1}(0.693)$

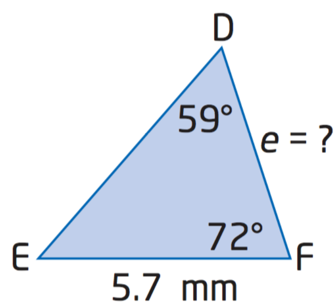
$\angle Y = 43.868\ldots$

$\angle Y = 43.9$

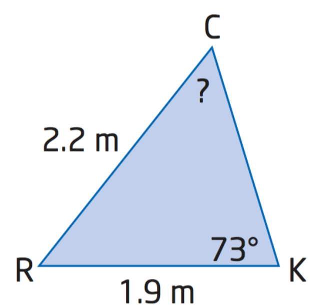
Opportunity to Learn

1. For each triangle, find the measure of the indicated angle or side.

a.



b.



2. In acute $\triangle ABC$, b = 15 m, c = 13 m, and $\angle B = 68°$.

   Draw a diagram and label the given information.

   Then, solve the triangle (find the measure of all the side lengths and all angles).

3. Mr. Ross measures the angle of elevation to the peak of a mountain as $35°$.

   Mr. Gordon is 1200 feet closer to the peak of the mountain on a straight level path.

   Mr. Gordon measures the angle of elevation as $42°$.

   How tall is the mountain?