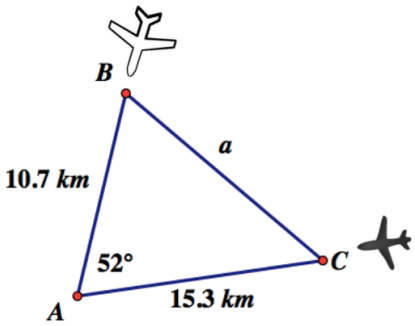
Developing the Cosine Law



Example 1

An air traffic controller at A is tracking two planes at B and C.

The planes are flying at the same altitude.

If the planes are less than 10 km apart, a warning must be issued to the pilots.

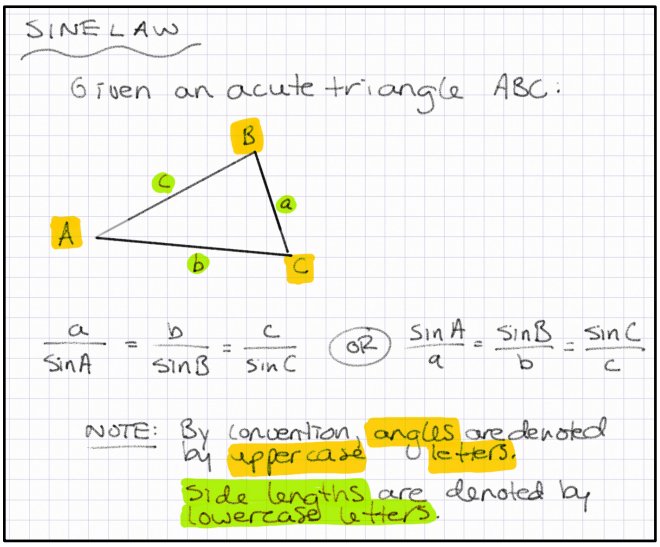
Should a warning be issued?

Example 2

Recall how in our most recent class, we dropped a perpendicular from the vertex of an acute triangle to the opposite side.

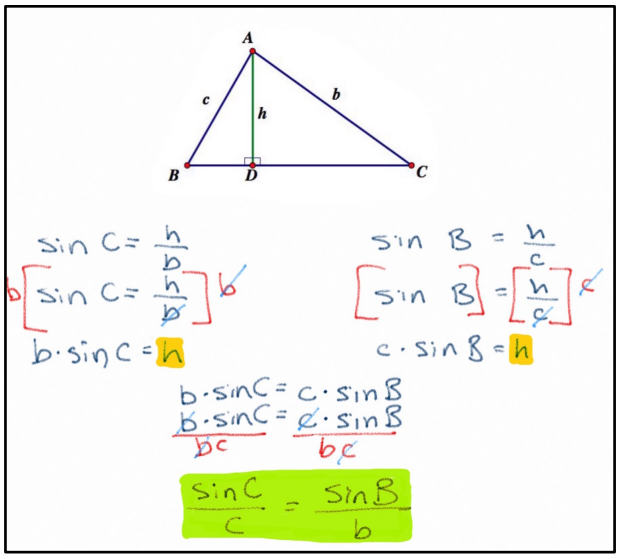
By using the *sine ratio* to connect the height of the triangle to the two right triangles that were created, the *Sine Law* was developed.

Formally…



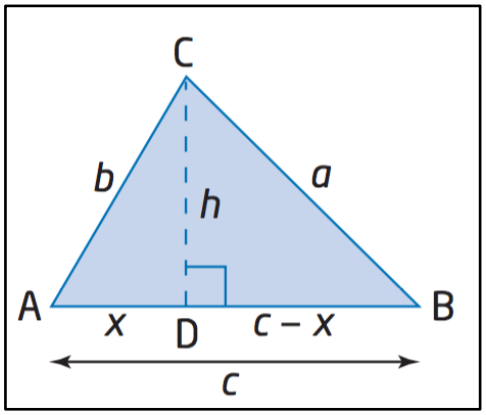
At left, consider $\triangle ADC$ and $\triangle BDC$.

Our goal is to develop a formula that only involves the measures of $\angle A$, $\angle B$, or $\angle C$ and side lengths a, b, and c.



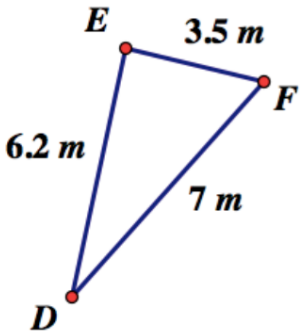
However…

… what if, after dropping the perpendicular, we had tried using the *cosine ratio* instead?



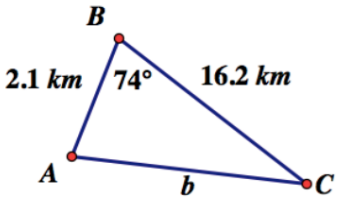
Example 3

What is the measure of angle D, to the nearest degree?



Opportunity to Learn

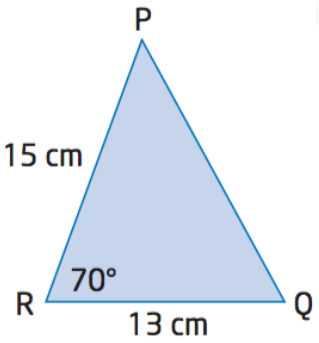
1. What is the length of b, to the nearest tenth of a kilometre?



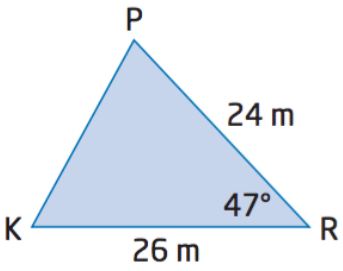
2. Solve each triangle. Round answers to the nearest unit, if necessary.

   NOTE: To solve a triangle means to find the measure of all sides and angles.

a)

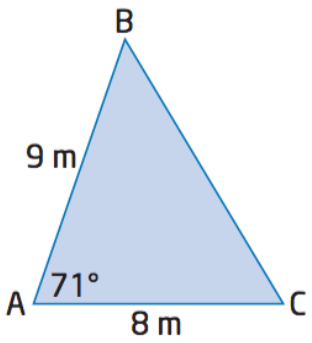


b)



2. (continued)

c)



3. Two observers who are 5 km apart simultaneously sight a small airplane flying between them.

   One observer measures a 51^{o} angle of elevation, while the other measures a 40.5^{o} angle of elevation.

   At what altitude is the airplane flying? Include a diagram with your solution.