## Quadratics — Maximizing Rectangular Area

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### Whiteboard Problem (Collaborative)

A farmer wants to build a rectangular enclosure against an existing barn. The barn forms one side of the rectangle, so the farmer only needs to fence three sides. The farmer has 100 meters of fencing. What dimensions maximize the area of the enclosure?

If you didn't write an equation to find the maximum area, which is possible, write an equation representing the area of the enclosure. That equation is a quadratic. Create a graph of the equation and label key points: y-intercept, x-intercepts, and vertex. State what each of those points means in this context.

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# What Do You Do When You Don't Know What to Do?

*a.k.a.* ***wdydwydkwtd***

You stare at a problem. Nothing happens. Your brain feels empty. Welcome — this is one of the most important moments in math, because what you do **next** is what actually grows you as a problem solver. Not knowing what to do isn't a failure. It's the start.

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## Try a strategy — *any* strategy

Mathematicians don't wait for inspiration. They reach into a toolbox and try a move. Pick something:

- **Draw it.** A diagram, picture, or model. Even a rough sketch can show you something the words were hiding.
- **Look for a pattern.** Try a few small cases. What repeats? What changes?
- **Guess and check.** A wrong guess is information. Two wrong guesses tell you which direction to move.
- **Make it simpler.** Solve an easier version first, then scale your idea up.
- **Make an organized list or table.** Structure beats chaos.
- **Work backwards.** Start at the goal and trace back.
- **Use logical reasoning.** What *must* be true? What *can't* be true?
- **Make assumptions** (and label them). Revisit them later.

## Switch representations

If algebra isn't clicking, try a graph. If a graph isn't clicking, try a table. If a table isn't clicking, try words. No single representation is “the math” — they're all windows into the same idea.

## Reflect — out loud or on paper

Ask yourself:

- *What do I actually know?*
- *What am I being asked to find?*
- *Have I seen something like this before?*
- *Is my approach getting me closer, or am I going in circles?*

If you've been stuck on the same approach for a while, that's a signal — not to give up, but to **switch**.

## Connect it to something you *do* know

Every problem connects to something. *What unit does this remind me of? What did we do last week that's similar?* Pulling in a related idea is often the unlock.

## Talk it through

Explaining where you're stuck — to a classmate, your teacher, even out loud to yourself — forces you to organize your thinking. Half the time, you find the next step before you finish the sentence.

## Check that your answer is reasonable

Once you have an answer, look back. Does it make sense? Is it in the right ballpark? Did you actually answer what was asked?

**The bottom line: not knowing what to do isn't the end of problem solving — it's the beginning.**
**Your job in that moment isn't to be brilliant. It's to pick a strategy, try it, watch what happens, and adjust.**

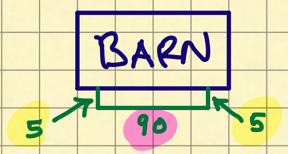
# Quadratics - Maximizing Rectangular Area

## WDYDWYDKWTD?

① **Draw it.** → "guess and check"

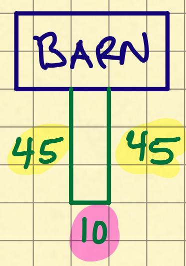
↳ I'm going to draw a few cases... not to scale.

"really skinny"



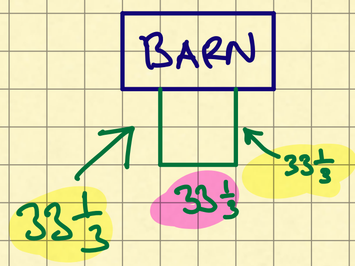

$A = lw$
$= 5(90)$
$= 450 \text{ m}^2$

"really skinny, other way"



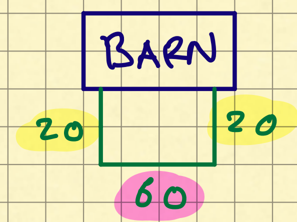

$A = lw$
$= 45(10)$

"square"




$A = lw$
$= (33\frac{1}{3})(33\frac{1}{3})$
$\doteq 1110.1$

"more rectangular"




$A = lw$
$= 20(60)$
$= 1200$

② **Look for a pattern.**

↳ long and skinny is no good (small area)

→ rectangular seems better

③ **Use a table.**

↳ from my examples above, there are two sides and one other side with whatever fencing is left over.

→ Hmm... let's generalize

Let $x$ be the length of the side that happens twice.

Since there are 100 metres of fencing to work with, that makes the length of the other side $100 - 2x$

| $l$ · | $w$ = | Area |
|---|---|---|
| $(x)$ | $(100 - 2x)$ = | $A$ |
| 5 | 90 | 450 |

| | | |
|---|---|---|
| 10 | 80 | 800 |
| 15 | 70 | 1050 |
| 20 | 60 | 1200 |
| 25 | 50 | 1250 |
| 30 | 40 | 1200 |
| 35 | 30 | 1050 |

Hmm... I notice that area is increasing up to a point, then decreasing again... that feels like a quadratic relationship.

④ Switch representations.

↳ I'll assume a quadratic relationship, then find the vertex.

$A = lw$

$A = x(100 - 2x)$

$A = 100x - 2x^2$

$0 = 100x - 2x^2$

$\dfrac{0}{-2} = \dfrac{100x - 2x^2}{-2}$

$0 = -50x + x^2$

$0 = x^2 - 50x$ ← re-arrange

$0 = x(x - 50)$ ← Factor out $x$.

$\therefore x = 0$ or $x = 50$

$\therefore h = \dfrac{0 + 50}{2}$

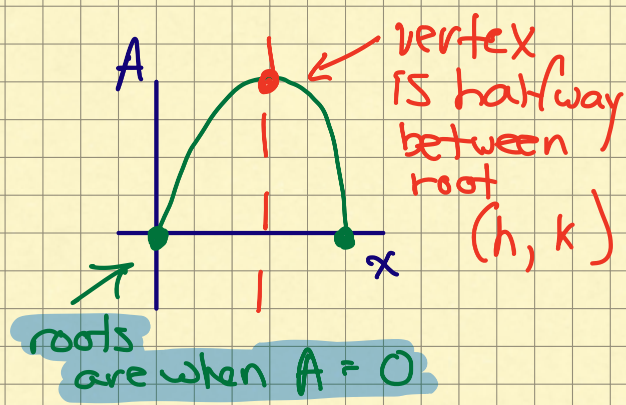
$= 25$

$\therefore k = x(100 - 2x)$

$= 25(50)$

$= 1250$

I'll find the vertex by averaging the roots... like this:




$\therefore$ vertex is at $(25, 1250)$

$\therefore$ the dimensions of the pen that maximize area are 25 metres by 50 metres.
I "got close" to this result by guessing and checking and then confirmed it using two representations (a table, and then using algebra).