Comparing and Describing Quadratic Relations

In a quadratic relation, when discussing a *transformation* of the parent relation $y = x^2$ there are three parameters:

$$y = a(x - h)^2 + k$$

Describe the impact of changing the values of each parameter.

a:

h:

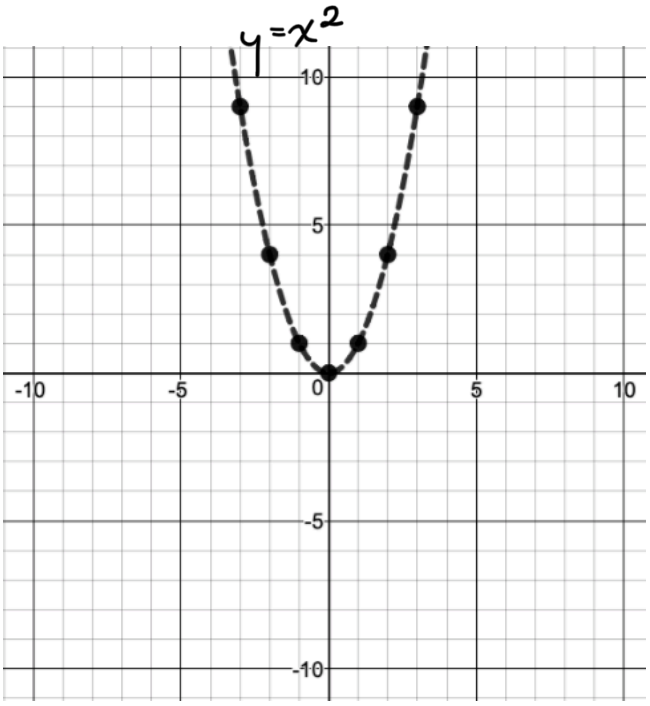
k:

Example 1

When the independent variable, x, is squared, and then three is subtracted, like so:

$$y = x^2 - 3$$

a) Why is that transformed graph shifted down, compared to $y = x^2$?
b) What are the characteristics of a quadratic relation when represented as a graph?
c) What are the characteristics of a quadratic relation when represented as a table?



$x^2 - 3$ first differences second differences

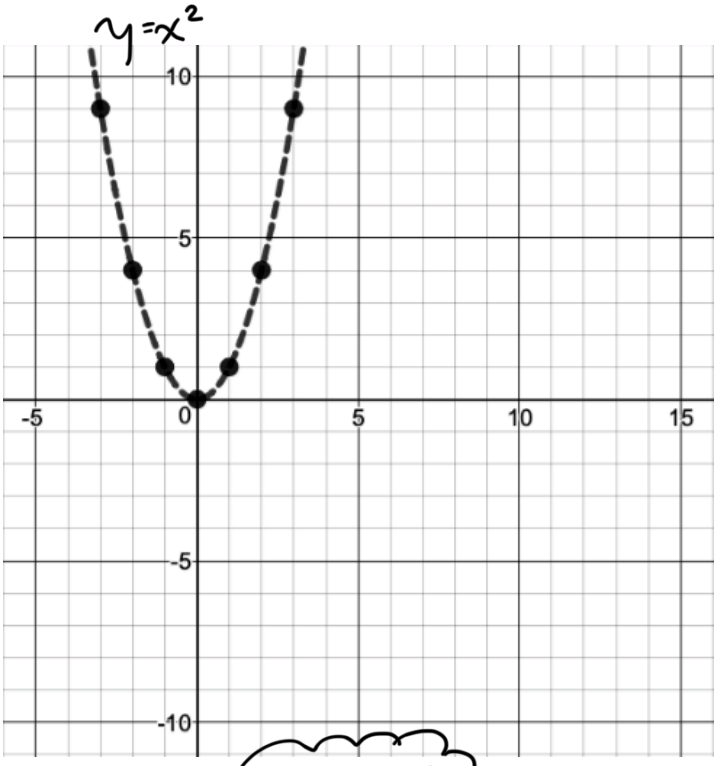
x	y	F.D.	S.D.
-3	$(-3)^2 - 3 = 6$	[redacted]	[redacted]
-2	$(-2)^2 - 3 = 1$	-5	[redacted]
-1	$(-1)^2 - 3 = -2$	-3	2
0			
1			
2			
3			

Example 2

When seven is subtracted from independent variable, x, and the result is squared, like so:

$$y = (x - 7)^2$$

a) Why is the transformed graph shifted to the right, compared to $y = x^2$?
b) What are the characteristics of a quadratic relation when represented as a graph?
c) What are the characteristics of a quadratic relation when represented as a table?



$(x - 7)^2$

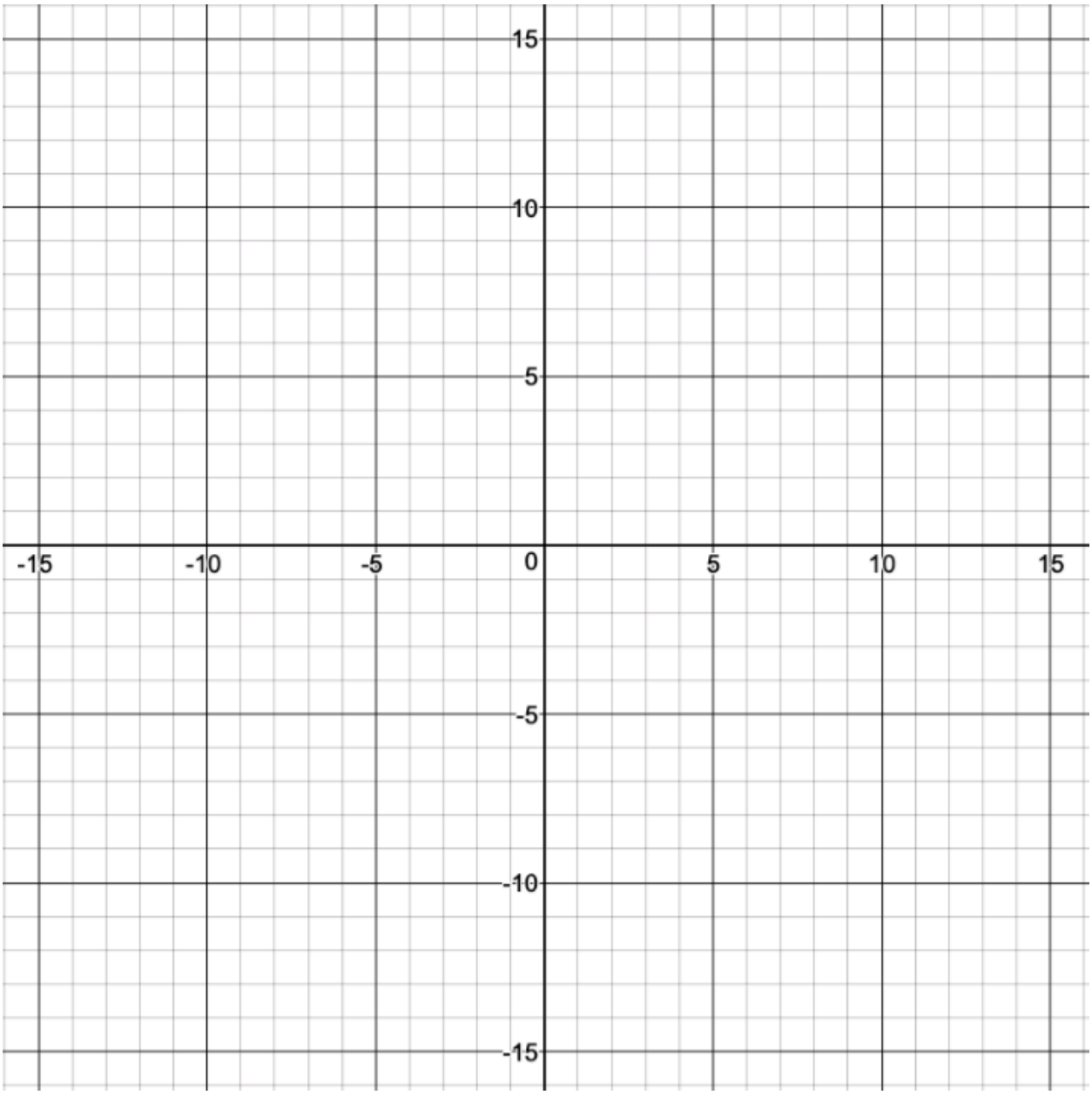
x	x - 7	y	F.D.	S.D.
4	-3	$(-3)^2 = 9$	[redacted]	
5				
6				
7				
8				
9				
10				

Example 3

Consider the first differences from the relations in examples 1 and 2. Do you see a pattern?

Let's try applying that pattern to plot, by hand, the following relations without resorting to making a table of values first:

a. $y = x^2 + 5$ b. $y = (x - 2)^2$ c. $y = -x^2 - 3$



Opportunity to Learn

Given what you have summarized about how the values of a, h, and k transform a quadratic in the form:

$$y = a(x - h)^2 + k$$

… try writing an equation for each quadratic described below.

a. The graph of $y = x^2$ is translated 6 units upward.

b. The graph of $y = x^2$ is translated 3 to the right.

c. The graph of $y = x^2$ is translated 7 units downward.

d. The graph of $y = x^2$ is translated 5 units to the left.

e. The graph of $y = x^2$ is translated 8 units downward, and 2 units to the right.

f. The graph of $y = x^2$ is translated 9 units upward, and 11 units to the right.

g. The graph of $y = x^2$ is translated 5 units downward, 4 units to the right, and reflected vertically so that it opens down (looks like an “n” rather than a “u”).