Comparing and Describing Quadratic Relations

In a quadratic relation, when discussing a *transformation* of the parent relation $y = x^2$ there are three parameters:

$$y = a(x-h)^2 + k$$ vertex form

Describe the impact of changing the values of each parameter.

a: Changes the shape of the parabola
changes the direction of opening (up/down)

h: changes the horizontal position of the vertex

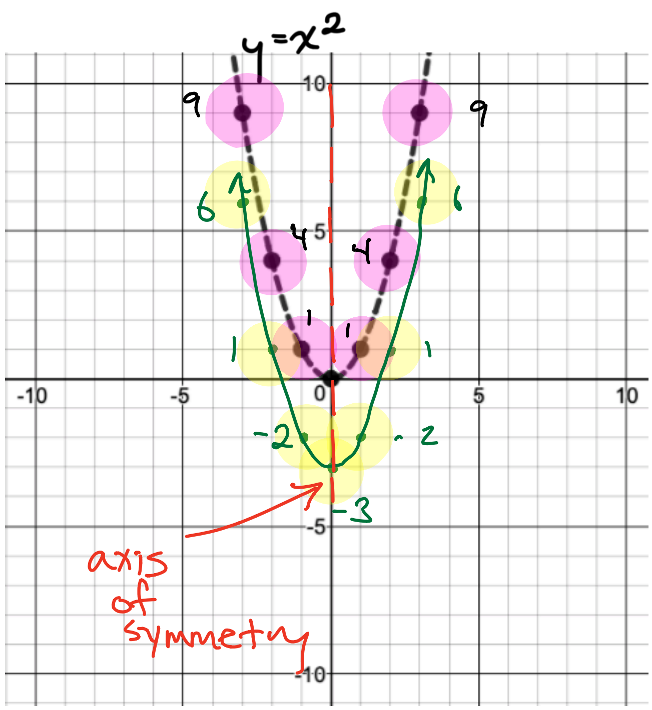
k: " " vertical " " " "

Example 1

When the independent variable, x, is squared, and then three is subtracted, like so:

$$y = x^2 - 3$$

a) Why is that transformed graph shifted down, compared to $y = x^2$?
b) What are the characteristics of a quadratic relation when represented as a graph?
c) What are the characteristics of a quadratic relation when represented as a table?



$x^2 - 3$ first differences second differences

x	y	F.D.	S.D.
-3	$(-3)^2-3=6$	[redacted]	[redacted]
-2	$(-2)^2-3=1$	-5	[redacted]
-1	$(-1)^2-3=-2$	-3	2
0	$(0)^2-3=-3$	-1	2
1	$(1)^2-3=-2$	1	2
2	$(2)^2-3=1$	3	2
3	$(3)^2-3=6$	5	2

a) Because the k value, -3, reduces the y-values by 3

b) curve w/ one vertex, symmetrical

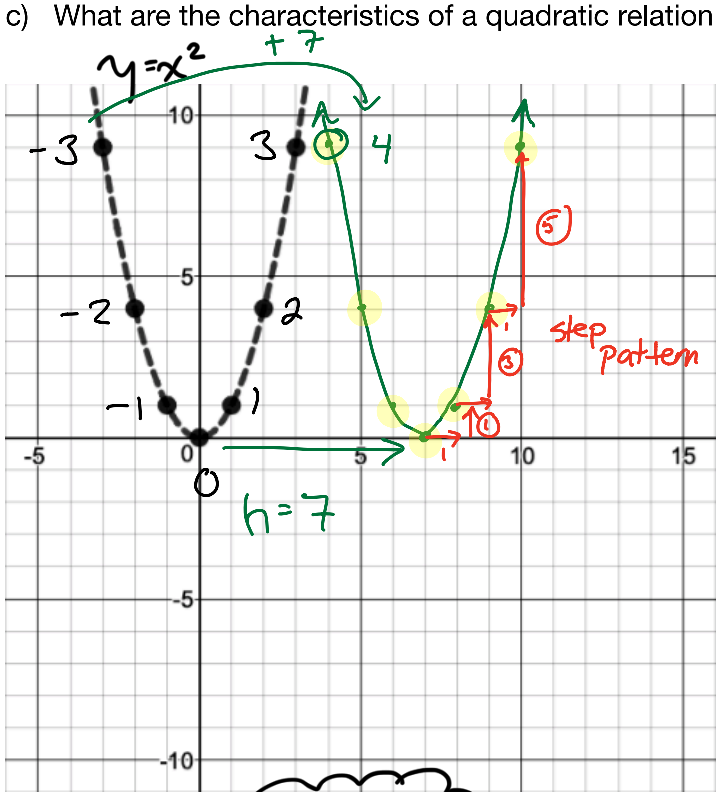
c) second differences are constant

Example 2

When seven is subtracted from independent variable, x, and the result is squared, like so:

$$y = (x - 7)^2$$

a) Why is the transformed graph shifted to the right, compared to $y = x^2$?
b) What are the characteristics of a quadratic relation when represented as a graph?
c) What are the characteristics of a quadratic relation when represented as a table?



a) Our inputs, the values for the independent variable, are 7 greater than with $y = x^2$

b) Same as example 1

c) second differences are all the same

$(x-7)^2$

x	x - 7	y	F.D.	S.D.
4	-3	$(-3)^2 = 9$	[redacted]	
5	-2	$(-2)^2 = 4$	-5	
6	-1	$(-1)^2 = 1$	-3	2
7	0	$(0)^2 = 0$	-1	2
8	1	$(1)^2 = 1$	1	2
9	2	$(2)^2 = 4$	3	2
10	3	$(3)^2 = 9$	5	2

Example 3

Consider the first differences from the relations in examples 1 and 2. Do you see a pattern?

1 , 3 , 5 change in y-values

Let's try applying that pattern to plot, by hand, the following relations without resorting to making a table of values first:

$y = a(x-h)^2 + k$

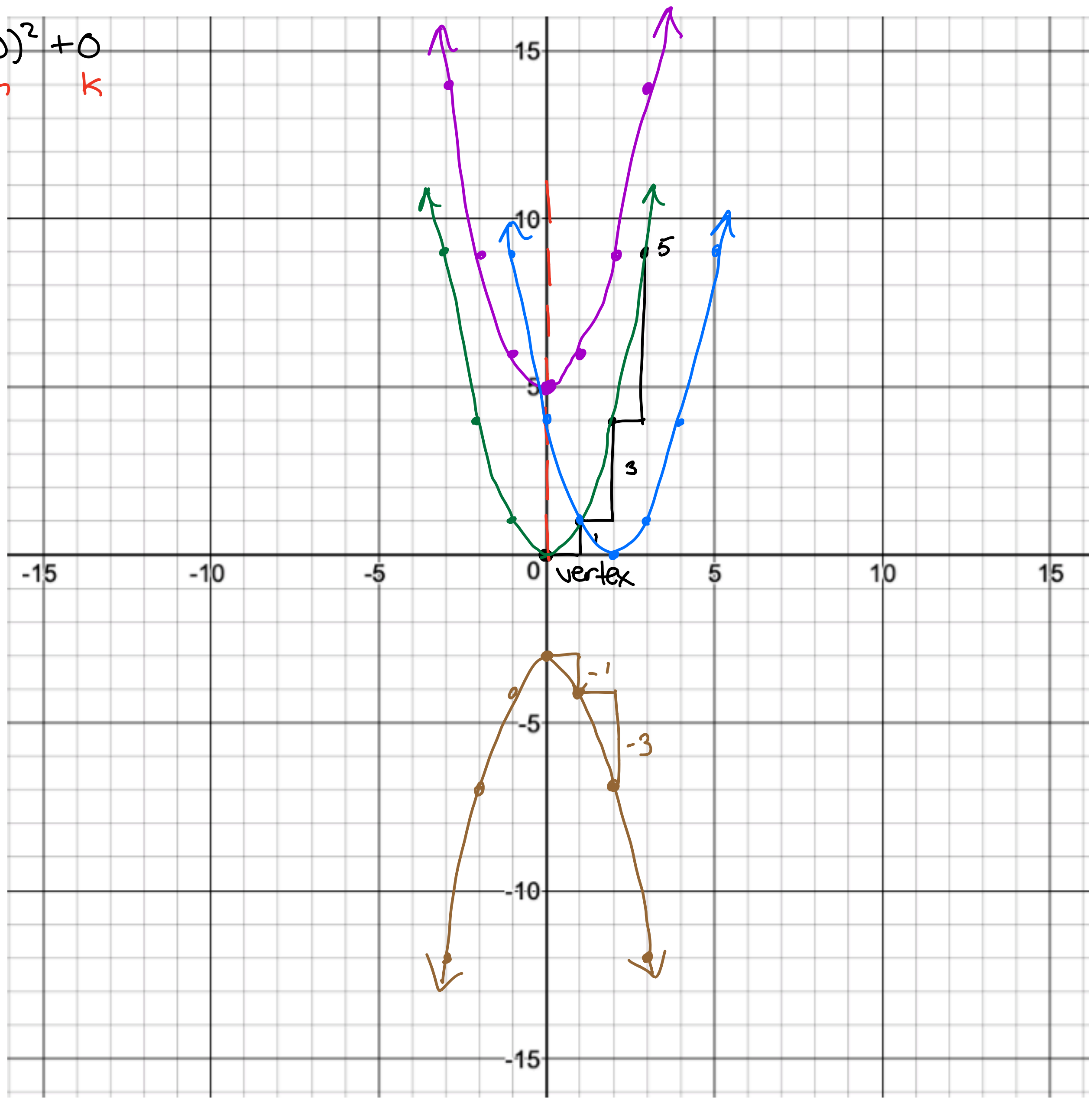
$y = x^2$

a. $y = x^2 + 5$ $k = 5$

b. $y = (x-2)^2$ $h = 2$

c. $y = -x^2 - 3$

$y = 1(x-0)^2 + 0$
a h k



Opportunity to Learn

Given what you have summarized about how the values of a, h, and k transform a quadratic in the form:

$$y = a(x - h)^2 + k$$

Use Desmos to check your answers!

… try writing an equation for each quadratic described below.

a. The graph of $y = x^2$ is translated 6 units upward. $y = x^2 + 6$

*Plot $y = x^2$
*Plot your answer
* Does new graph match the description?

b. The graph of $y = x^2$ is translated 3 to the right.

c. The graph of $y = x^2$ is translated 7 units downward.

d. The graph of $y = x^2$ is translated 5 units to the left.

e. The graph of $y = x^2$ is translated 8 units downward, and 2 units to the right.

f. The graph of $y = x^2$ is translated 9 units upward, and 11 units to the right.

g. The graph of $y = x^2$ is translated 5 units downward, 4 units to the right, and reflected vertically so that it opens down (looks like an “n” rather than a “u”).