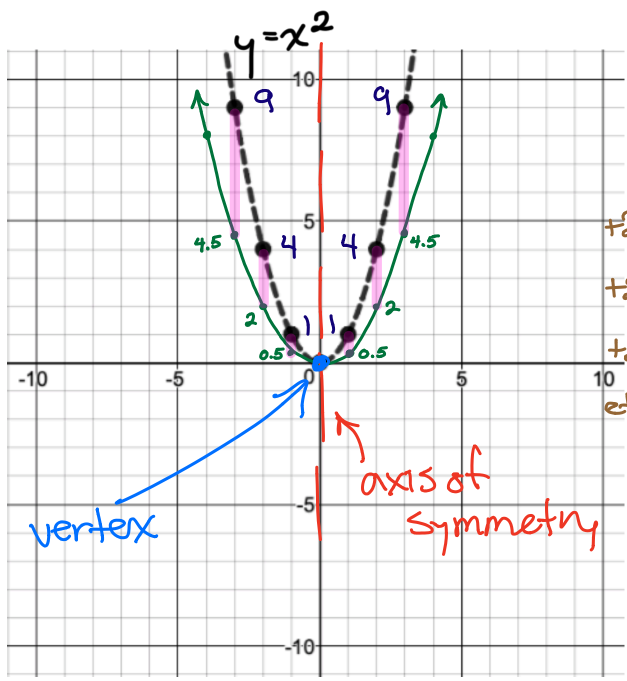
Comparing and Describing Quadratic Relations, Part II

Example 1

When the independent variable, x, is squared, and the result is multiplied by 0.5, like so:

$$y = 0.5x^2$$

a) Why does the new graph appear to be wider, or compressed, compared to $y = x^2$?
b) What are the characteristics of a quadratic relation when represented as a graph?
c) What are the characteristics of a quadratic relation when represented as a table?



$0.5x^2$ first differences second diff.

x	y	F.D.	S.D.
-6	$0.5(-6)^2 = 18$	[redacted]	[redacted]
-4	$0.5(-4) = 8$	-10	[redacted]
-2	$0.5(-2) = 2$	-6	4
0	$0.5(0) = 0$	-2	4
2	$0.5(2) = 2$	2	4
4	$0.5(4) = 8$	6	4
6	$0.5(8) = 18$	10	4

+2 +2 +2 etc

3 $0.5(3)^2 = 4.5$

a) compared to $y = x^2$, the graph of $y = 0.5x^2$ is compressed, because all of the y-values are half as large

b) a compressed quadratic relation has all of the same characteristics as the parent quadratic (symmetrical, has a vertex, et cetera)

c) represented as a table, a quadratic relation will always have second differences that are constant (the same)

Example 2

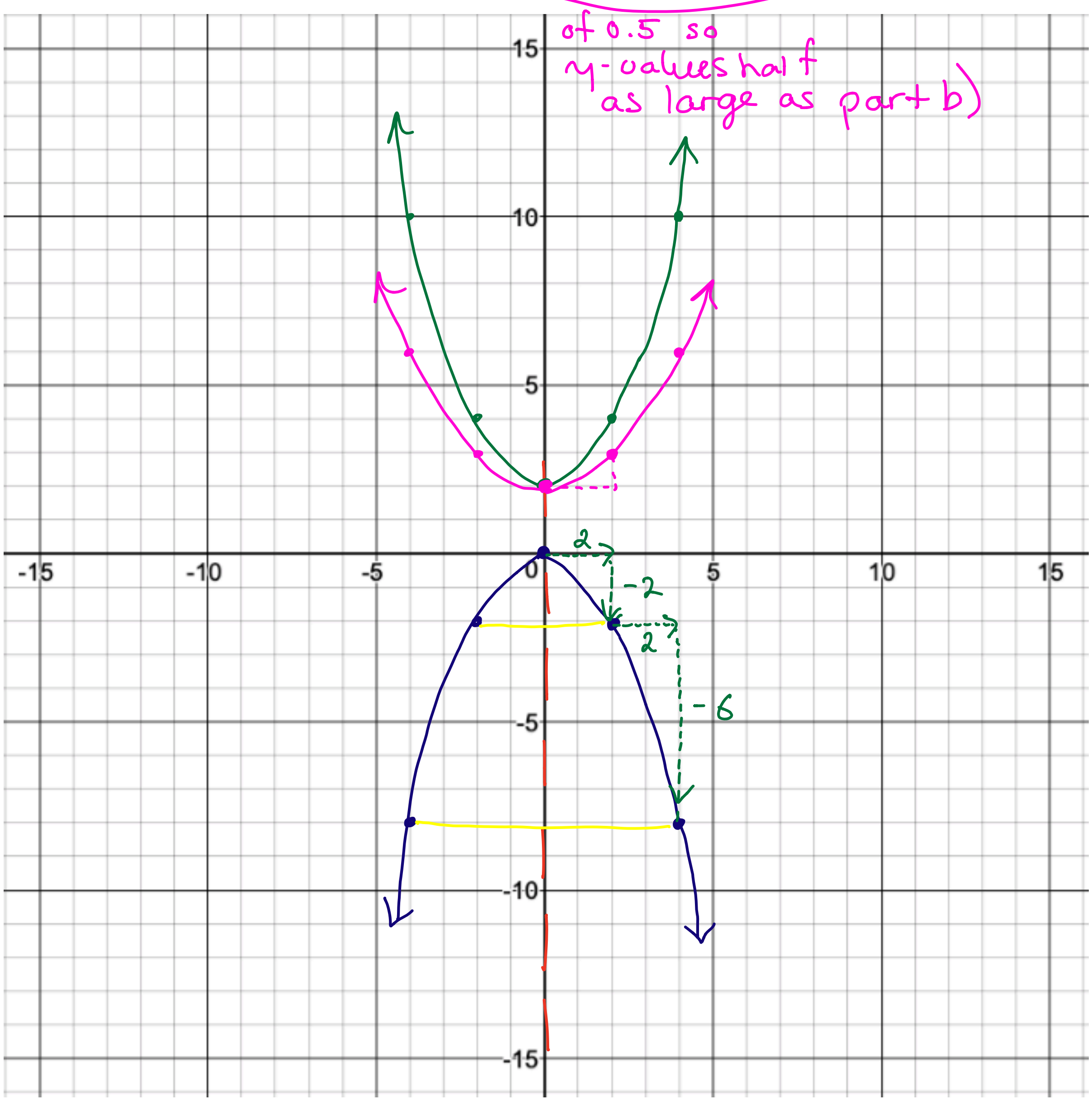
Consider the first differences from the relations in example 1. Do you see a pattern?

- first differences follow same pattern of decrease and increase (we can use this to plot a quadratic quickly)

Let's try applying that pattern to plot, by hand, the following relations without resorting to making a table of values first:

a. $y = -0.5x^2$ b. $y = 0.5x^2 + 2$ c. $y = 0.25x^2 + 2$

0.25 is half of 0.5 so y-values half as large as part b)

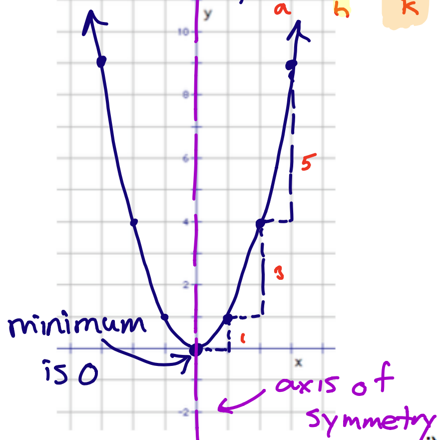


Example 3

Plot each of the following relations and analyze the relation as indicated:

a) Graph the relation $y = x^2$.

$y = 1(x-0)^2 + 0$
a h k



i) Direction of opening?
up

ii) Vertex co-ordinates?
(0, 0)

iii) Eq'n of axis of symmetry?
$x = 0$

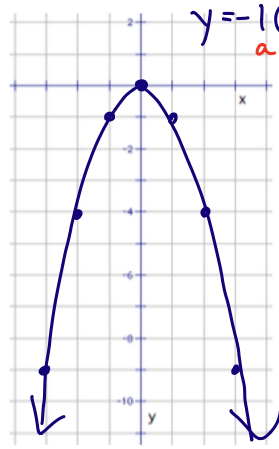
iv) Max/min value is?
min = 0 max = ∞

v) Values x may take?
$-\infty$ to $+\infty$

vi) Values y may take?
0 to $+\infty$

b) Graph the relation $y = -x^2$

$y = -1(x-0)^2 + 0$
a h k



i) Direction of opening?
down

ii) Vertex co-ordinates?
(0, 0)

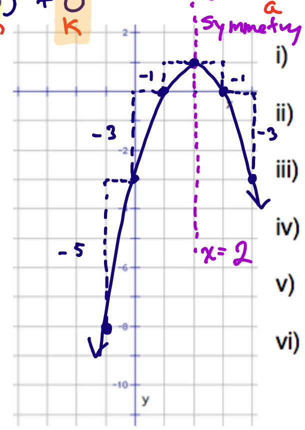
iii) Eq'n of axis of symmetry?
$x = 0$

iv) Max/min value is?
min = $-\infty$ max = 0

v) Values x may take?
$-\infty$ to $+\infty$

vi) Values y may take?
$-\infty$ to 0

c) Graph the relation $y = -(x-2)^2 + 1$
a h k



i) Direction of opening?
down

ii) Vertex co-ordinates?
(2, 1)

iii) Eq'n of axis of symmetry?
$x = 2$ $x = h$

iv) Max/min value is?
max = 1

v) Values x may take?
$-\infty$ to $+\infty$

vi) Values y may take?
$-\infty$ to 1

Opportunity to Learn

Part A

Given what you have summarized about how the values of a, h, and k transform a quadratic in the form:

$$y = a(x-h)^2 + k$$

… try writing an equation for each quadratic described below.

a. A parabola with its vertex at $(2, 3)$, opening up, with no vertical stretch.
b. A parabola with its vertex at $(-3, 0)$, opening down, with a vertical stretch of factor 2.