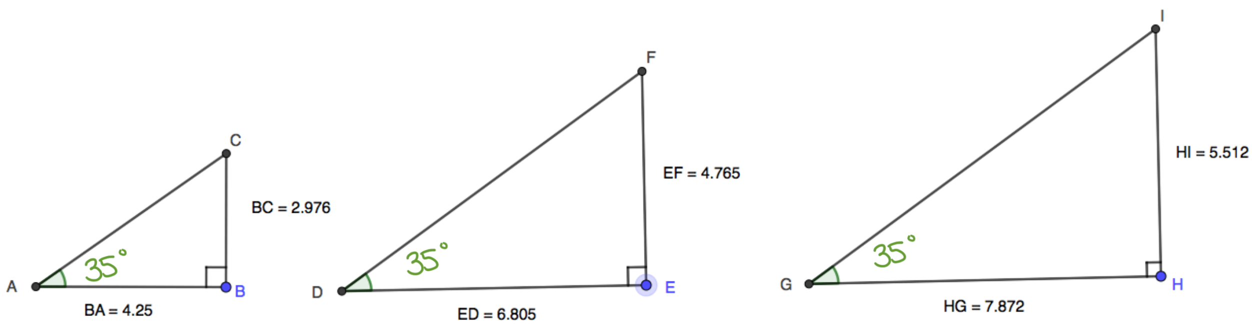
Applying the Tangent Ratio

To save a bit of time, what we figured out in our last class is referred to as the **tangent ratio**.

That is, given a right triangle of any size, the slope always matches a given angle.

Or, given a right triangle of any size, the ratio of the opposite and adjacent side lengths always matches a given angle.

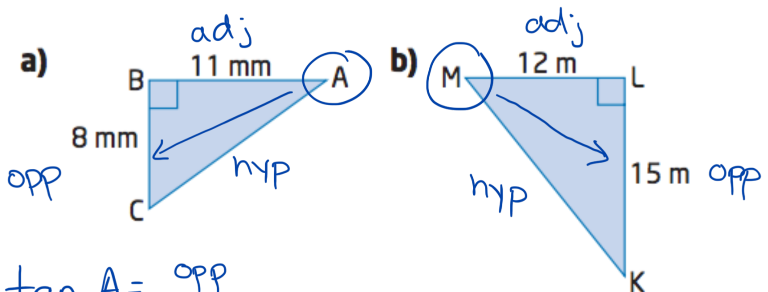
For example, a slope (or tangent ratio) of 0.700 always corresponds to a 35° angle.



$\frac{BC}{BA} = \frac{2.976}{4.25} = 0.7$ $\qquad \frac{EF}{ED} = \frac{4.765}{6.805} = 0.7$ $\qquad \frac{HI}{HG} = \frac{5.512}{7.872} = 0.7$

Example 1

Find the measures of both acute angles in each triangle, to the nearest full degree.



$\tan A = \frac{opp}{adj}$

$= \frac{8}{11}$

$\doteq 0.727$

(from the table) $\angle A = 36^\circ$

(a ratio of 0.727 always corresponds to an angle of 36°)

$\angle C = 180^\circ - 90^\circ - 36^\circ$

$= 54^\circ$

$\tan M = \frac{15}{12}$

$= 1.25$

* $\angle M =$ between 51° and 52°

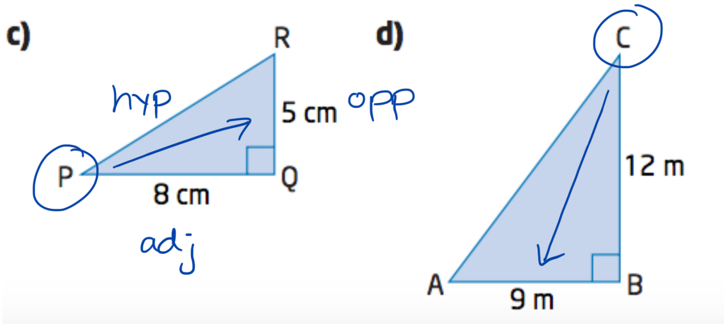
* 1.25 is closer to 1.235 than to 1.280

$\therefore \angle M \doteq 51$

$\angle K \doteq 180^\circ - 90^\circ - 51^\circ$

$\doteq 39^\circ$

6	0.105	51	1.235
7	0.123	52	1.280
8	0.141	53	1.327
9	0.158	54	1.376
10	0.176	55	1.428
11	0.194	56	1.483
12	0.213	57	1.540
13	0.231	58	1.600
14	0.249	59	1.664
15	0.268	60	1.732
16	0.287	61	1.804
17	0.306	62	1.881
18	0.325	63	1.963
19	0.344	64	2.050
20	0.364	65	2.145
21	0.384	66	2.246
22	0.404	67	2.356
23	0.424	68	2.475
24	0.445	69	2.605
25	0.466	70	2.747
26	0.488	71	2.904
27	0.510	72	3.078
28	0.532	73	3.271
29	0.554	74	3.487
30	0.577	75	3.732
31	0.601	76	4.011
32	0.625	77	4.331
33	0.649	78	4.705
34	0.675	79	5.145
35	0.700	80	5.671
36	0.727	81	6.314
37	0.754	82	7.115
38	0.781	83	8.144



$\tan P = \frac{5}{8}$

$= 0.625$

$\angle P = 32°$

or... on calculator...

$\angle P = \tan^{-1}(0.625)$

$= 32°$

$\angle R = 180° - 90° - 32°$

$= 58°$

$\tan C = \frac{9}{12}$

$= 0.75$

* $\angle C$ = between 36° and 37°

* 0.75 is closer to 0.754 than 0.727

$\therefore \angle C = 37°$

$\angle A = 180° - 90° - 37°$

$= 53°$

The Tangent Ratios

$\tan^{-1}$: Ratio → Angle; tan: Angle → Ratio

Angle	Ratio	Angle	Ratio
0	0.000	45	1.000
1	0.017	46	1.036
2	0.035	47	1.072
3	0.052	48	1.111
4	0.070	49	1.150
5	0.087	50	1.192
6	0.105	51	1.235
7	0.123	52	1.280
8	0.141	53	1.327
9	0.158	54	1.376
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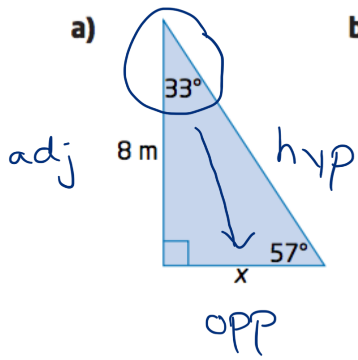
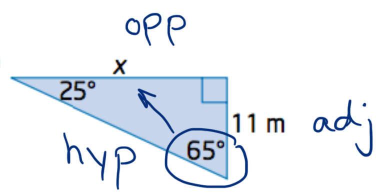
27	0.510	72	3.078
28	0.532	73	3.271
29	0.554	74	3.487
30	0.577	75	3.732
31	0.601	76	4.011
32	0.625	77	4.331
33	0.649	78	4.705
34	0.675	79	5.145
35	0.700	80	5.671
36	0.727	81	6.314
37	0.754	82	7.115
38	0.781	83	8.144
39	0.810	84	9.514
40	0.839	85	11.430
41	0.869	86	14.301
42	0.900	87	19.081
43	0.933	88	28.636
44	0.966	89	57.290
45	1.000	90	undefined

Example 2

Find the length of x, to the nearest tenth of a metre.

• pick an angle to look from
• aim to have the unknown in numerator if possible

a) 

b) 

$\tan 33° = \frac{x}{8}$

$8\left[\tan 33°\right] = \left[\frac{x}{\cancel{8}}\right]\cancel{8}$

$8(0.649) \doteq x$

$5.2 \doteq x$

$\tan 65° = \frac{x}{11}$

$11\left[\tan 65°\right] = \left[\frac{x}{\cancel{11}}\right]\cancel{11}$

$11(2.145) \doteq x$

$23.6 \doteq x$

The Tangent Ratios

Angle → tan → Ratio; Ratio → $\tan^{-1}$ → Angle

Angle	Ratio	Angle	Ratio
0	0.000	45	1.000
1	0.017	46	1.036
2	0.035	47	1.072
3	0.052	48	1.111
4	0.070	49	1.150
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44	0.966	89	57.290
45	1.000	90	undefined

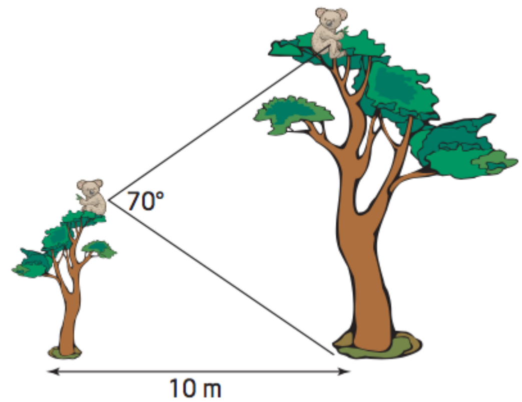
Opportunity to Learn

1. Two koalas sit at the top of two eucalyptus trees.

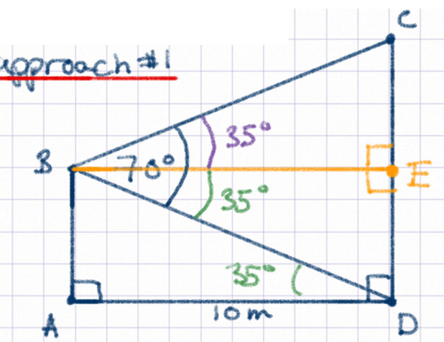
   The first tree is exactly half as tall as the second tree.

   How high off the ground is each koala?

   NOTE: Assume that the 10m side at the bottom of the diagram extends only to the vertex at which there is a 70° angle.



approach #1



$\because 2\overline{AB} = \overline{CD}$ then BE bisects $\angle CBD$

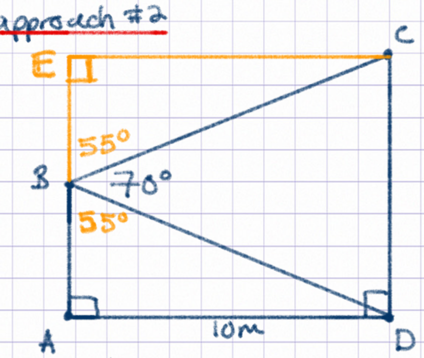
$\therefore \angle CBE = \angle DBE = 35^\circ$.

and $\angle BEC = \angle BED = 90^\circ$

$\therefore BE \parallel AD$

$\therefore \angle DBE = \angle ADB = 35^\circ$ (PLT-Z)

approach #2



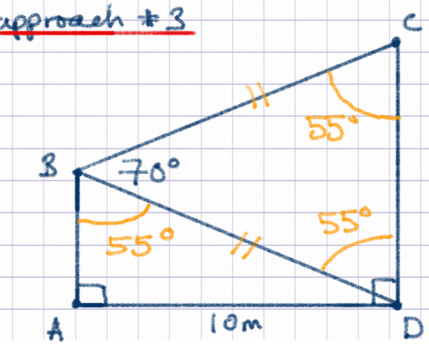
Extend AB to E. Draw segment EC such that $\angle CEB = 90^\circ$.

$\because 2\overline{AB} = \overline{CD}$ then $\triangle ABD \cong \triangle EBC$

$\therefore \angle EBC = \angle ABD = \frac{180^\circ - 70^\circ}{2}$

$= 55^\circ$ (by SAT)

approach #3



$\because 2\overline{AB} = \overline{CD}$ then $\overline{BD} = \overline{BC}$

$\therefore \triangle CBD$ is isosceles

$\therefore \angle BCD = \angle BDC = \frac{180 - 70^\circ}{2}$

$= 55^\circ$ (by ITT)

$\because CD \parallel AB$ $\angle ABD = \angle CDB = 55^\circ$ (PLT-Z)

building from approach #1

$\tan 35° = \frac{AB}{10}$

$10[\tan 35°] = \left[\frac{AB}{\cancel{10}}\right]\cancel{10}$

$10(\tan 35°) = AB$

$10(0.700) \doteq AB$

$7 \doteq AB$

$CD = 2AB$
$= 2(7)$
$= 14$

∴ the first koala is 7 m off the ground and the second koala is 14 m off the ground

building from approach #2, #3

* by SATT, determine that $\angle ADB = 180° - 90° - 55°$
$= 35°$

* then proceed exactly as shown at left for approach #1.

* or... go directly to:

$\tan 55° = \frac{10}{AB}$

$AB[\tan 55°] = \left[\frac{10}{\cancel{AB}}\right]\cancel{AB}$

$AB(\tan 55°) = 10$

$\frac{AB(\cancel{\tan 55°})}{\cancel{\tan 55°}} = \frac{10}{\tan 55°}$

$AB = \frac{10}{\tan 55°}$

$AB \doteq \frac{10}{1.428}$

$AB \doteq 7$

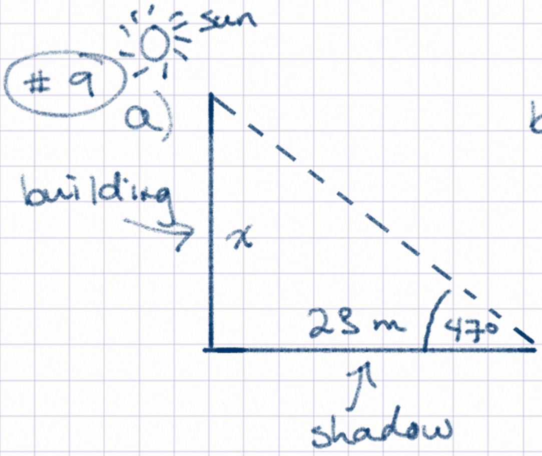
$CD = 2AB$
$= 2(7)$
$= 14$

∴ the first koala is 7 m off the ground and the second koala is 14 m off the ground

2. Complete [questions 9 to 12 from the file at this link](http://tinyurl.com/y3uutckl) (`http://tinyurl.com/y3uutckl`). Final answers are on the second page of the file. Use a separate sheet of paper to complete those questions.

#9

a)



b) Let x be the height of the building in metres.

$$\tan 47^\circ = \frac{x}{23}$$

$$23\left[\tan 47^\circ\right] = \left[\frac{x}{\cancel{23}}\right]\cancel{23}$$

$$23(\tan 47^\circ) = x$$

$$23(1.072) \doteq x$$

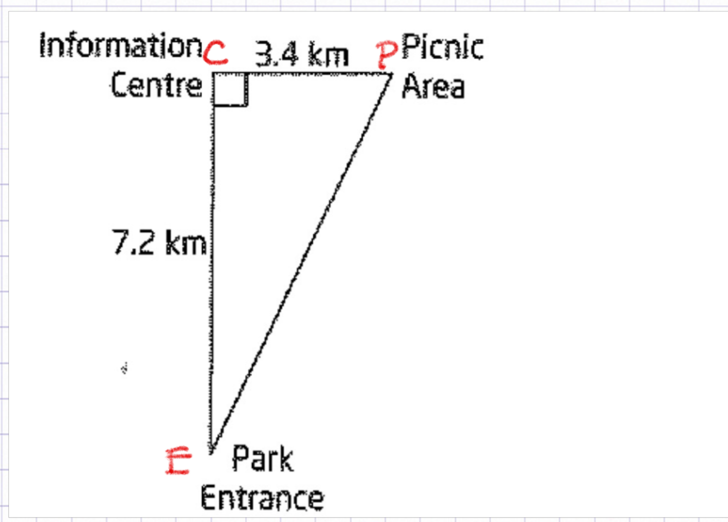
$$24.656 \doteq x$$

$$24.7 \doteq x$$

∴ rounded to the nearest tenth of a metre, the height of the building is 24.7 m.

#10

a)



we are trying to find ∠CEP)

$$\tan\angle CEP = \frac{3.4}{7.2}$$

$$\tan\angle CEP \doteq 0.472$$

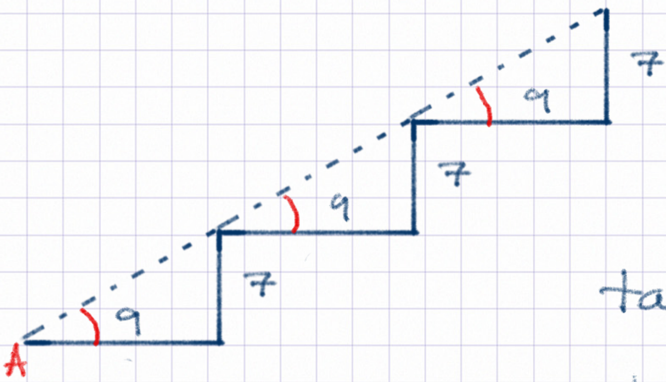
$$\angle CEP \doteq 25^\circ$$

(consulting the table and rounding down to 25° since 0.472 is closer to 0.466 than 0.488).

b) We assume the segment EP will be the shortest distance to travel only if there are no obstacles, that the ground is perfectly (or mostly) flat compared to walking along another path from point E to P.

#11

Slope of $\frac{7}{9}$ implies a rise of 7 units and a run of 9 units, since slope is $\frac{\text{rise}}{\text{run}}$.



Let A be the angle the stairs make with the horizontal.

$\tan A = \frac{opp}{adj}$

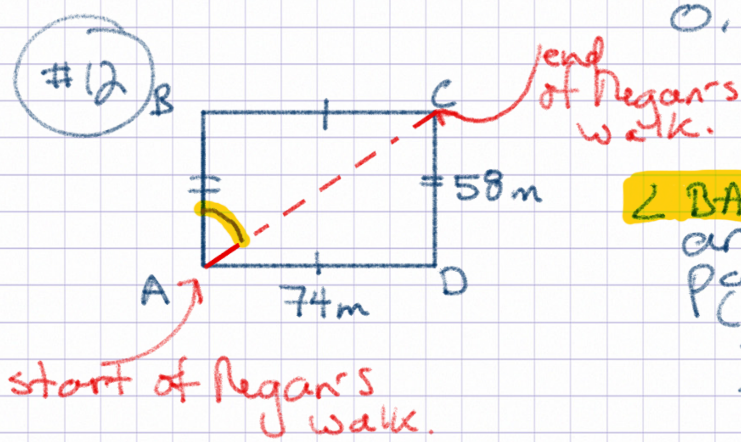
$\tan A = \frac{7}{9}$

$\tan A \doteq 0.778$

$\angle A \doteq 38°$

(consulting table and rounding up since 0.778 is closer to 0.781 than 0.754.)

#12



$\angle BAC$ represents the angle formed by the path of her walk (compared to the shorter side of the triangle).

$\tan \angle BAC = \frac{74}{58}$

$\tan \angle BAC \doteq 1.276$

$\angle BAC \doteq 52°$ (rounding up)

$\therefore$ Megan walked at a 52° angle compared to the shorter side.