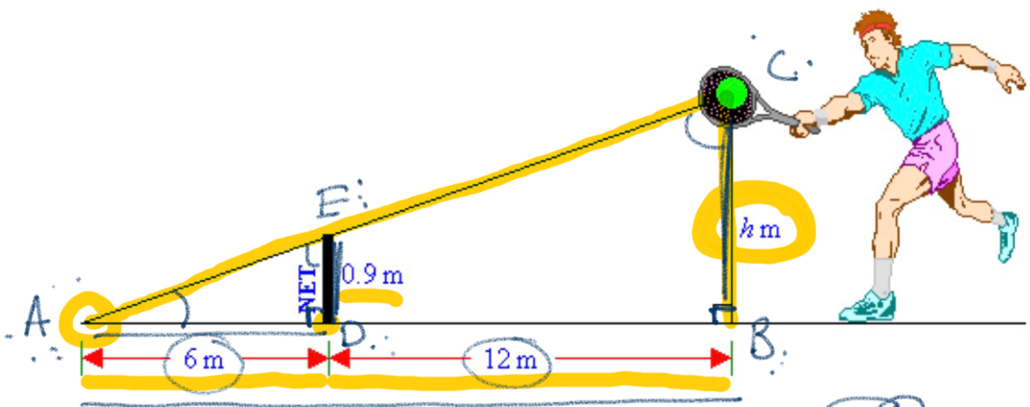
Applications of Similar Figures

Example 1

Find the height in metres, h, at which the tennis ball must be hit so that it will just pass over the net and land 6 metres away from the base of the net.



$\angle BAC = \angle DAE$ (same angle!) AAA

$\angle ABC = \angle ADE$ (both 90°, assuming net is $\perp$ floor).

$\angle ACB = \angle AED$ (SATT)

$\triangle ABC \sim \triangle ADE$.

"similar to"

$$\frac{BC}{DE} = \frac{AB}{AD}$$

"since" ∴ we know the △'s are similar, we can set up the proportion (ratios of corresponding sides in similar △'s are equal.

$$\frac{h}{0.9} = \frac{18}{6} \times 3$$

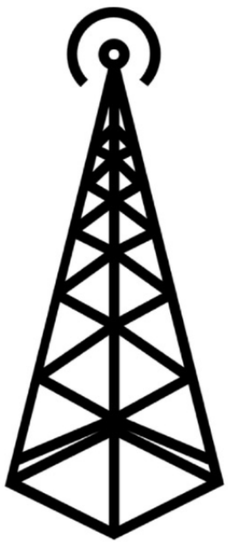
$h = 0.9(3)$

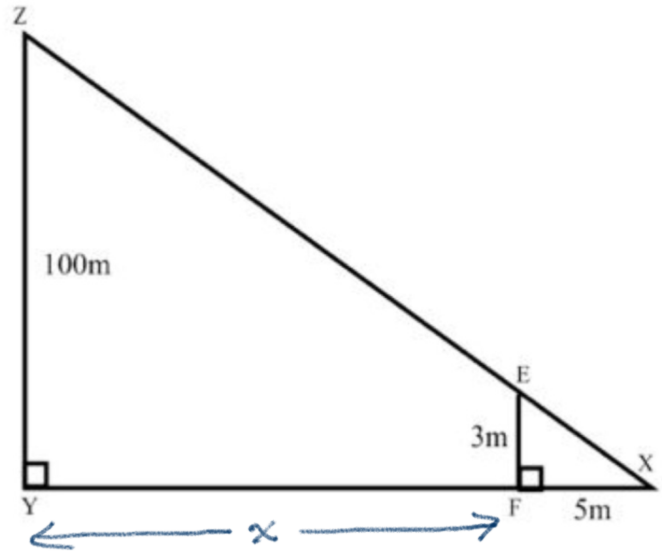
$h = 2.7$ m

∴ the tennis ball should be struck 2.7m above the ground.

Example 2

Maria, at point F, used a 3 m surveyor's pole to sight from the ground to the top of a 100m tower. Based on her diagram, how far was she from the tower? How do we know the triangles are similar?





Is $\triangle ZYX \sim \triangle EFX$?

$\angle ZXY = \angle EXY$ (literally the same angle)

$\angle ZYX = \angle EFX$ (both $90°$, given)

$\angle YZX = \angle FEX$ (by SATT).

$\therefore \triangle ZYX \sim \triangle EFX$.

Now we can use equal ratios of corresponding sides:

Let x be the length of YF.

$$\frac{ZY}{EF} = \frac{YX}{FX}$$

$$\frac{100}{3} = \frac{x+5}{5}$$

$$15\left[\frac{100}{3}\right] = \left[\frac{x+5}{5}\right]15$$

$$5(100) = 3(x+5)$$

$$500 = 3x + 15$$

$$-15 \qquad -15$$

$$\frac{485}{3} = \frac{3x}{3}$$

$$161.7 \doteq x$$

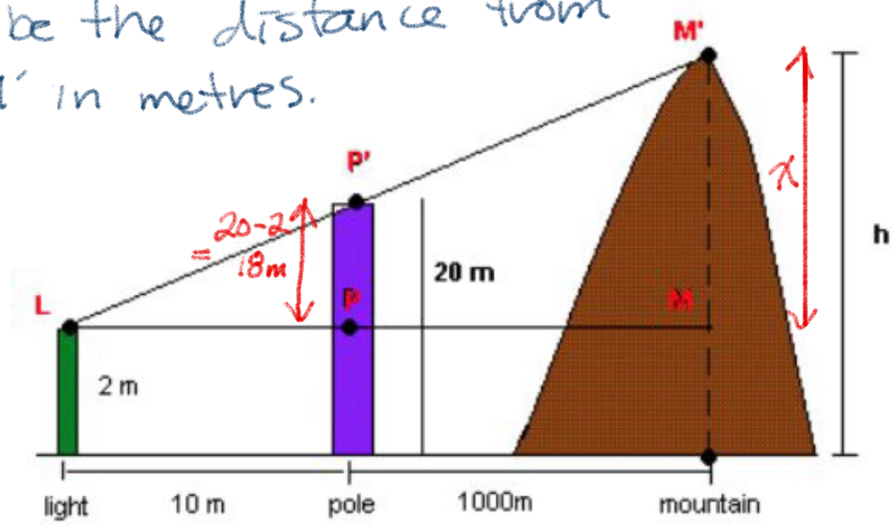
$\therefore$ Maria is about 162 m away.

Opportunity to Learn

1. A research team wishes to determine the altitude of a mountain as follows: They use a light source at L, mounted on a structure of height 2 meters, to shine a beam of light through the top of a pole P' through the top of the mountain M'. The height of the pole is 20 meters. The distance between the altitude of the mountain and the pole is 1000 meters. The distance between the pole and the laser is 10 meters. We assume that the light source mount, the pole and the altitude of the mountain are on the same plane (the ground is completely flat).

   Find the altitude, h, of the mountain in metres.

Let x be the distance from M to M′ in metres.



Is $\triangle LPP' \sim \triangle LMM'$?

$\angle M'LM = \angle P'LP$ (same)
$\angle LMM' = \angle LPP'$ (both 90° assumed)
$\angle MM'L = \angle PP'L$ (SATT)
$\therefore \triangle LPP' \sim \triangle LMM'$.

Now: $\dfrac{MM'}{PP'} = \dfrac{LM}{LP}$

$\dfrac{x}{18} = \dfrac{10+1000}{10}$

$\dfrac{x}{18} = \dfrac{1010}{10}$

$18 \cdot \cancel{10}\left[\dfrac{x}{\cancel{18}}\right] = \left[\dfrac{1010}{\cancel{10}}\right] 18 \cdot \cancel{10}$

$10x = 1010(18)$

$\dfrac{10x}{10} = \dfrac{1010(18)}{10}$

$x = 1818$

Then: $h = x + 2$
$h = 1820$

$\therefore$ the mountain has an altitude of 1820 m.

2. Mme. Dalrymple wants to measure the height of the LCS Academic Block building, but she does not have the tools available to measure the height directly. She noticed that there is a tree located in front of the building so she decided to use her geometry superpowers to determine the building's height.

   She measured the distance between the tree and the building and found that it is 30 m. She stood in front of the tree and started backing up until she could see the top edge of the building from above the tree top. She marked her place and measured it from the tree. It was 5 m.

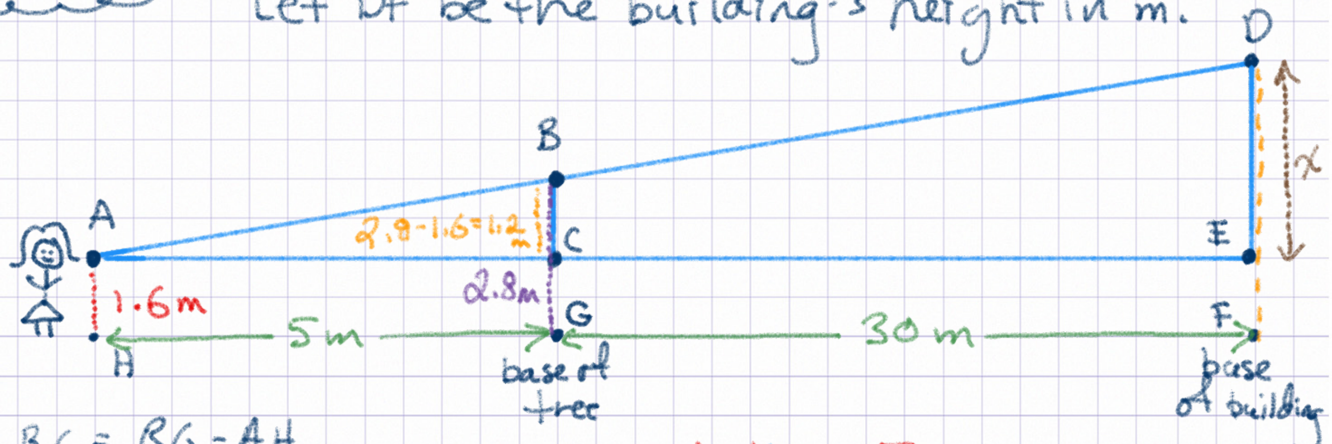
   Knowing that the tree's height is 2.8m and that the height of Mme. Dalrymple's eyes above the ground is 1.6m, apply your own math superpowers to determine the height of the school.

diagram not to scale

Let AH be the distance from the ground to Madame's eyes in m.

Let BG be the tree's height in m.

Let DF be the building's height in m.



$BC = BG - AH$
$= 2.8 - 1.6$
$= 1.2$

① Is $\triangle ABC \sim \triangle ADE$?

$\angle BAC = \angle DAE$ (given)
$\angle ACB = \angle AED$ (assumed 90°)
$\angle ABC = \angle ADE$ (SATT).
$\therefore \triangle ABC \sim \triangle ADE$.

② Then: $\frac{AE}{AC} = \frac{DE}{BC}$

$\frac{35}{5} = \frac{x}{1.2}$

$(1.2)(\cancel{5})\left[\frac{35}{\cancel{5}}\right] = \left[\frac{x}{\cancel{1.2}}\right](5)(\cancel{1.2})$

$(1.2)(35) = (5)(x)$

$\frac{42}{5} = \frac{5x}{5}$

$8.4 = x$

③ Then: $DF = DE + EF$ EF=AH
$= x + AH$
$= 8.4 + 1.6$
$= 10m$

3. For each part below:

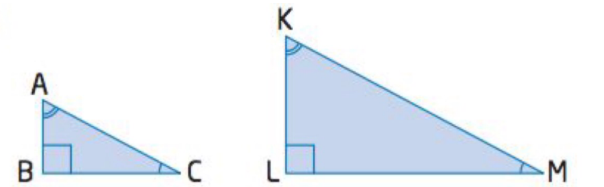
- name the similar triangles
  e.g.: $\triangle ABC \sim \triangle KLM$

- identify the equal angles with letters denoting each angle in corresponding order
  e.g.: $\angle ABC = \angle KLM$, $\angle BCA = \angle LMK$, etc.

- identify corresponding sides and write the equivalent ratios of side lengths
  e.g.:

$$\frac{AB}{KL} = \frac{BC}{LM} = \frac{CA}{MK}$$

a)



$\triangle ABC \sim \triangle KLM \because$ $\angle ABC = \angle KLM$, $\angle BCA = \angle LMK$, $\angle CAB = \angle MKL$

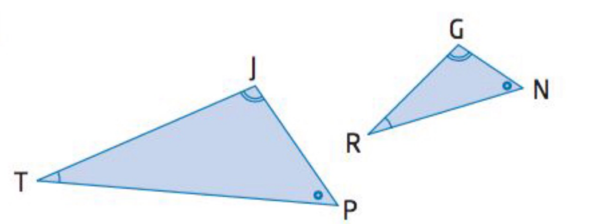
"triangle" "is similar to" "since" "angle"

Then:

we know these sides correspond because they start and end at equal angles in the figure.

$$\frac{AB}{KL} = \frac{BC}{LM} = \frac{CA}{MK}.$$

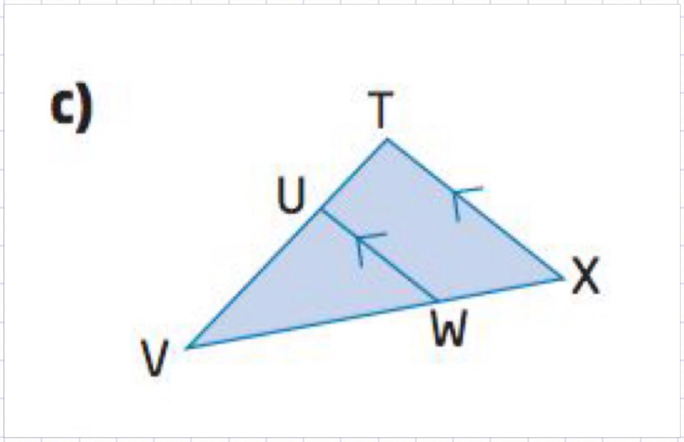
b)



$\triangle TPJ \sim \triangle RNG \because$ $\angle JTP = \angle GRN$, $\angle TPJ = \angle RNG$, $\angle PJT = \angle NGR$

Then:

$$\frac{TP}{RN} = \frac{PJ}{NG} = \frac{JT}{GR}$$



$\triangle UVW \sim \triangle TVX$

$\therefore$

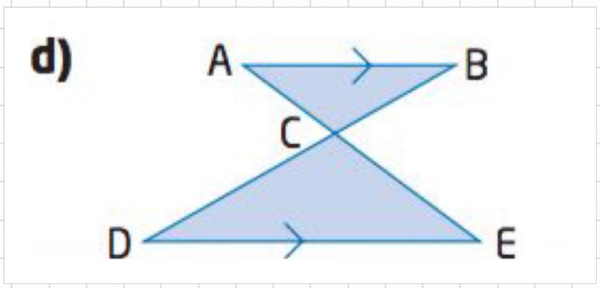
$\angle UVW = \angle TVX$ (same angle)

$\angle VWU = \angle VXT$ ("F" pattern, corresponding angles)

$\angle WUV = \angle XTV$

(SATT)

Then: $\dfrac{UV}{TV} = \dfrac{VW}{VX} = \dfrac{WU}{XT}$



$\triangle ABC \sim \triangle CDE$

$\therefore$

$\angle ACB = \angle DCE$ (OAT)

$\angle ABC = \angle CDE$ ("Z" pattern, alternate angles)

$\angle CAB = \angle CED$ (SATT)

Then: $\dfrac{AB}{ED} = \dfrac{AC}{EC} = \dfrac{BC}{DC}$