Applications of the Primary Trigonometric Ratios

Example 1

A scuba driver swam north at 1.5 m/s across a current running from east to west at 2.0 m/s.

She swam for 3 minutes and then surfaced.

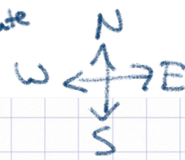
a. Draw a diagram showing where the dive boat will pick her up relative to where she dove.

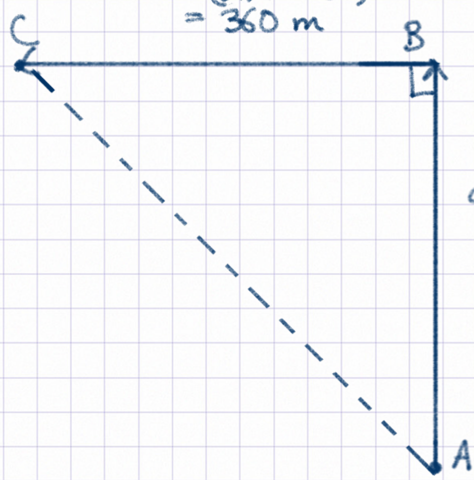
b. How far did she travel?

$d = s \cdot t$ ← 60 seconds in a minute

$= (2)(3 \cdot 60)$

$= 360$ m





$d = s \cdot t$ ← 60 seconds in a minute

$= (1.5)(3 \cdot 60)$

$= 270$ m

starting position

$AC^2 = AB^2 + BC^2$

$AC^2 = 270^2 + 360^2$

$AC^2 = 202500$

$\sqrt{AC^2} = \sqrt{202500}$

$AC = 450$

$\therefore$ She travelled 450 m on her dive.

Without an east to west current, the scuba diver would simply travel 270 m north.

With the current pushing her east to west, she will end up at point C having travelled along line segment AC.

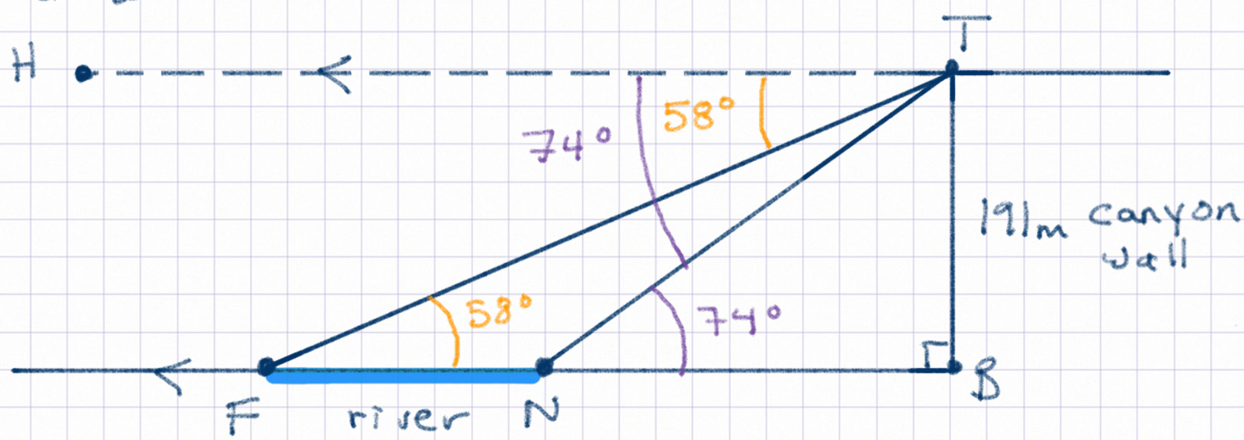
Example 2

From the top of a canyon, the angle of depression to the far side of the river is 58 degrees, and the angle of depression to the near side of the river is 74 degrees. The depth of the canyon is 191 m.

What is the width of the river at the bottom of the canyon?

Round to the nearest tenth of a meter.

Assuming a "sheer" canyon wall that meets ground at 90°.



$\angle FTH = \angle TFB = 58°$ (Z pattern)

$\angle NTH = \angle TNB = 74°$ (Z pattern)

need FN, the width of the river.

$FN = FB - NB$

to get FB:

$\tan TFB = \frac{TB}{FB}$

$\tan 58° = \frac{191}{FB}$

$FB\left[\tan 58°\right] = \left[\frac{191}{\cancel{FB}}\right]\cancel{FB}$

$\frac{FB(\tan 58°)}{\tan 58°} = \frac{191}{\tan 58°}$

$FB \doteq 119.35$

to get NB

$\tan TNB = \frac{TB}{NB}$

$\tan 74° = \frac{191}{NB}$

$NB\left[\tan 74°\right] = \left[\frac{191}{\cancel{NB}}\right]\cancel{NB}$

$\frac{NB(\tan 74°)}{\tan 74°} = \frac{191}{\tan 74}$

$NB \doteq 54.77$

to get FN

$FN = FB - NB$

$= 119.35 - 54.77$

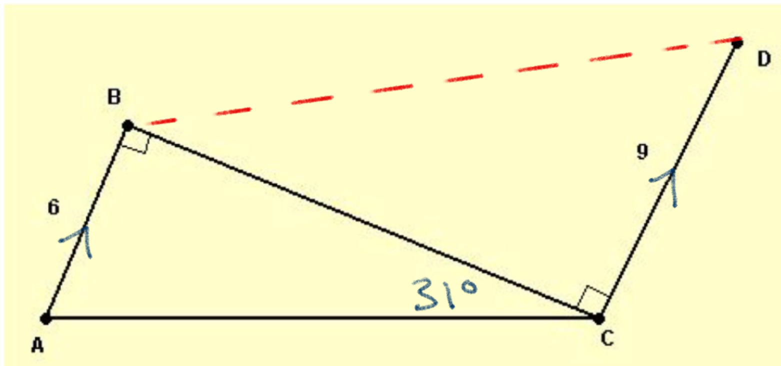
$\doteq 64.6$

$\therefore$ the river is about 64.6m wide

Opportunity to Learn

1. In the figure below AB and CD are perpendicular to BC and the size of angle ACB is 31°.

   Find the length of line segment BD.



$\because AB \perp BC$ and $CD \perp BC$ then $AB \parallel CD$

To get BD we first need to get BC.

$\tan \angle ACB = \dfrac{AD}{BC}$ $\dfrac{opp}{adj}$

$BC\left[\tan 31^\circ\right] = \left[\dfrac{6}{BC}\right] BC$

$\dfrac{BC(\cancel{\tan 31^\circ})}{\cancel{\tan 31^\circ}} = \dfrac{6}{\tan 31^\circ}$

$BC \doteq 9.9857$

Then:

$BD^2 = BC^2 + CD^2$

$BD^2 = (9.9857)^2 + 9^2$

$\sqrt{BD^2} = \sqrt{(9.9857)^2 + 9^2}$

$BD \doteq 13.4$

$\therefore$ BD is about 13.4 units long.

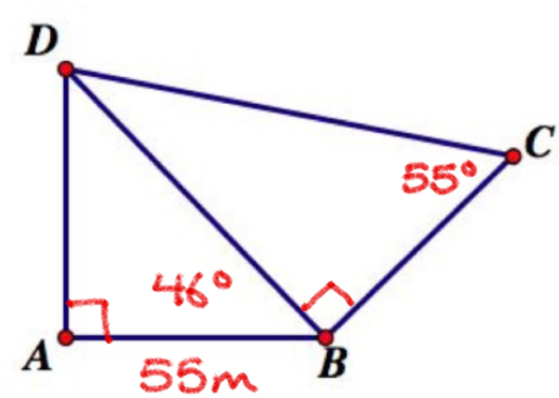
2. $\angle DAB$ and $\angle DBC$ are both 90°.

$\angle DBA$ is 46°. $\angle DCB$ is 55°.

$\overline{AB}$ is 55 m.

What is $\overline{DC}$?

Show your work, please.



To get DC, we first need some other side length in $\triangle DBC$.

We can get DB by using the cosine ratio in $\triangle DAB$.

To get DB:

$$\cos DBA = \frac{AB}{DB} \quad \frac{(adj)}{(hyp)}$$

$$\cos 46^\circ = \frac{55}{DB}$$

$$DB\left[\cos 46^\circ\right] = \left[\frac{55}{\cancel{DB}}\right]\cancel{DB}$$

$$\frac{DB(\cancel{\cos 46^\circ})}{\cancel{\cos 46^\circ}} = \frac{55}{\cos 46^\circ}$$

$$DB \doteq 79.176$$

To get DC:

$$\sin DCB = \frac{DB}{DC} \quad \frac{(opp)}{(hyp)}$$

$$\sin 55^\circ \doteq \frac{79.176}{DC}$$

$$DC\left[\sin 55^\circ\right] = \left[\frac{79.176}{\cancel{DC}}\right]\cancel{DC}$$

$$\frac{DC(\cancel{\sin 55^\circ})}{\cancel{\sin 55^\circ}} = \frac{79.176}{\sin 55^\circ}$$

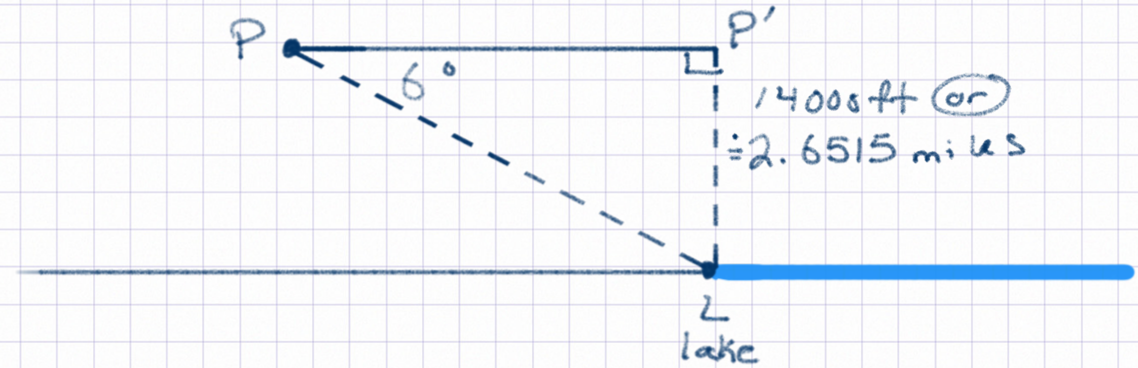
$$DC \doteq 96.7$$

$\therefore$ the length of line segment DC is about 96.7m

3. A plane is flying at a constant altitude of 14,000 ft and a constant speed of 500 mph. The angle of depression from the plane to a lake is 6 degrees. To the nearest minute, how much time will pass before the plane is directly over the lake?

1 mile = 5280 feet

$$\frac{14000\text{ft}}{5280\text{ft/m}} \doteq 2.6515 \text{ miles}$$



Need PP' then we can use the speed to determine how long it will take the plane to reach the lake.

So...

$$\tan P = \frac{P'L}{PP'} \quad \frac{(\text{opp})}{(\text{adj})}$$

$$\tan 6^\circ = \frac{2.6515}{PP'}$$

$$PP'\left[\tan 6^\circ\right] = \left[\frac{2.6515}{\cancel{PP'}}\right]\cancel{PP'}$$

$$\frac{PP'(\tan 6^\circ)}{\tan 6^\circ} = \frac{2.6515}{\tan 6^\circ}$$

$$PP' \doteq 25.23 \text{ miles}$$

Now:

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{PP'}{\text{plane's speed}} = \frac{25.23}{500} = 0.05046 \text{ hours}$$

Finally:

$$0.05046 \text{ hour} \times \frac{60 \text{ min}}{1 \text{ hour}} \doteq 3.0 \text{ minutes}$$

∴ it will take about 3 minutes for the plane to reach the lake.